CALIBRATION AND ESTIMATION OF PROGRAMMING MODELS FOR AGRIGULTURAL SUPPLY ANALYSIS

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List of Acronyms

ABIAS	Absolute Bias
CAP	Common Agricultural Policy
CAPRI	Common Agricultural Policy Regional Impact
CAP-STRAT	Common Agricultural Policy Strategy for Regions, Agriculture and Trade
CES	Constant Elasticity of Substitution
CGE	Computable General Equilibrium
CONOPT	Constrained Optimisation (Solver)
ECAM	European Community Agricultural Model
EU	European Union
FAIR	Food and Agricultural Industries Research (EU-program)
GAMS	General Algebraic Modelling System
GCE	Generalised Cross Entropy
GME	Generalised Maximum Entropy
GME-D	Data Constrained Generalised Maximum Entropy
LP	Linear Program
ME	Maximum Entropy
MSE	Mean Square Error
NLP	Non-Linear Program
NQ	Normalised Quadratic
PMP	Positive Mathematical Programming
QP	Quadratic Programming
RMSE	Root Mean Square Error
SABIAS	Sum of Absolute Biases
SPEP	Symmetric Positive Equilibrium Problem
SRMSE	Sum of Root Mean Square Errors
WATSIM	World Agricultural Trade Simulation Model

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CALIBRATION AND ESTIMATION OF PROGRAMMING MODELS FOR AGRIGULTURAL SUPPLY ANALYSIS

1 Introduction

Quantitative models of multi-output, multi-input supply behaviour in agriculture typically belong to one of two main methodological types: either programming models or dual systems of supply and input demand equations. The former determine input allocation to various production activities using an explicit optimisation, the latter constitute analytical solutions to economic optimisation models. Maintained economic hypotheses and objectives do not necessarily have to differ between these types.[1] However, in empirical reality the structure and specification procedures are clearly distinguished: A programming model is chosen when the analyst sees the need to explicitly model complex technological or political constraints under which behavioural functions cannot be derived easily or at all. This generally comes at the cost of lacking statistical estimation and validation for the whole model. Dual equation systems, on the other hand, allow to apply well established econometric techniques to base the parametric specification on observed supply and input demand decisions of agricultural producers. This choice limits the model's complexity and potentially oversimplifies for the purpose of a differentiated analysis.

During the last decade both type of methodological approaches seemingly moved a little closer to each other. CHAMBERS and JUST (1989) developed a dual supply model specification with explicit allocation of fixed factors. This allowed to overcome a previous deficiency for modelling agricultural crop supply by incorporating land constraints and the observable decision variable 'land allocated to production activities'. It also provided a useful framework to model the European policy instrument 'hectare premium' distinct from product price effects (GUYOMARD et al. 1996; MORO and SCHOKAI 1999; ADENÄUER 1999). Nevertheless, additional constraints cannot easily be incorporated and the choice of functional form is restricted due to analytical limitations. From the programming side, HOWITT (1995a) presented 'Positive Mathematical Programming' (PMP) which allows to calibrate models to observed behaviour of a base year. PMP established itself as the dominant approach for the specification of programming models

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[1] Often, programming models are characterised as 'normative' in general, because they use an explicit optimisation. This neither reflects the original meaning (HENRICHSMEYER et al. 1993: 33-40) nor is it a very useful distinction. The objective of normative analysis is to say 'what should be' and in this respect farm- or regional *planning* models qualify for this category. Programming models designed to explain or project behaviour do not. An integratable 'positive' dual supply system could just as well be used as an explicit optimisation model for simulation and yield the exact same results.

designed for policy analysis (for example: HOWITT and GARDNER 1986; HOUSE 1987; KASNAKOGLU and BAUER 1988; HORNER et al. 1992; SCHMITZ 1994; ARFINI and PARIS 1995; BARKAOUI and BUTAULT 1999; CYPRIS 2000; GRAINDORGE et al. 2001; HELMING et al. 2001; PARIS et al. 2000). The incorporation of several observations employing an econometric criterion was generally made possible by PARIS and HOWITT (1998) and put to work for a cross sectional data set by HECKELEI and BRITZ (2000). However, the theoretical base of this approach is weak or at least veiled.

This thesis aims at further moving the two methodological approaches closer together. It shall advance the knowledge on the calibration and estimation of programming models in theory and application. It is structured by three main chapters: Chapter 2 reviews PMP as a model calibration approach and discusses different ways parameters have been specified in this context. Then, it is shown that PMP is not well suited for the estimation of programming models based on multiple observations and a general alternative is introduced which employs first order necessary conditions as equations to be estimated. The remainder of the chapter takes a closer look at various constrained optimisation models for crop supply specifications. This is supposed to provide the structure of estimable systems of equations under different empirically relevant rationalisations of agricultural supply behaviour. The objective of chapter 3 is to prove the functionality of the approach with Monte Carlo simulation results for three exemplary optimisation models. In addition, approaches using prior information exploit the potential of maximum entropy techniques in this context and address the problem of small sample sizes often confronted by differentiated modelling exercises. Chapter 4 is devoted to a 'real world' application and in fact provides the context in which the topic of this thesis has been discovered. It introduces concept and explorative application of the EU-wide regionalised sector model CAPRI (Common Agricultural Policy Regional Impact) for which the author takes responsibility together with a European network of researchers. Specific consideration is given to the specification of regional supply modules still characterised by a PMP motivated estimation approach. A critical evaluation of this application in the light of the results from chapters 2 and 3 and an overview on currently explored modifications is added. Chapter 5 summarises the results of the study and points at promising directions for further research.

2 Positive Mathematical Programming and Alternatives

2.1 Introduction to Chapter 2

Positive Mathematical Programming (PMP) is a methodology developed to calibrate Programming models to observed quantities by using information contained in dual variables of calibration constraints to specify appropriate non-linear objective functions. The standard approach can be attributed to HOWITT, but the technique has been employed by a series of pragmatic, policy oriented modelling exercises long before HOWITT (1995a) presented a more rigorous treatment of the methodological background (for example: HOUSE 1987; KASNAKOGLU and BAUER 1988; BAUER and KASNAKOGLU 1990; HORNER et al. 1992; SCHMITZ 1994; for more references see HOWITT 1995a).[2] The attractiveness of the exact calibration property combined with the promise to constrain the simulation behaviour of the models less severely than previously employed approaches ('bounds') lead to a significant interest and continued implementation of this approach (in the area of agricultural sector modelling see for example ARFINI 1996; CYPRIS 1996, 2000, GOHIN and CHANTREUIL 1999; ARFINI and PARIS 1995; BARKAOUI and BUTAULT 1999, BARKAOUI et al. 2001; GRAINDORGE et al. 2001, HELMING et al. 2001).

PARIS and HOWITT (1998) introduce an econometric criterion for the specification of PMP models and claim that they offer an approach which resolves an old methodological controversy in agricultural supply analysis by creating a 'continuum of analysis from mathematical programming to traditional econometrics' (p.125). However, the experience with PMP approaches based on more than one observation is very small (WILD 2000; HECKELEI and BRITZ 2000) and the theoretical motivation beyond the calibration capabilities of the approach is practically non-existent. Also, the theoretical relationship between econometric supply models with explicit allocation of fixed inputs – mainly based on duality theory - and PMP has not been systematically elaborated on. Are there other useful approaches in this area? Is the estimation of programming models really an alternative to the estimation of models with behavioural functions? What are possible advantages and disadvantages of both approaches?

This chapter intends to answer a few of these questions. The main objective is to build some theoretical ground for the estimation of programming models, i.e. laying a foundation for the incorporation of more than one data point in the specification of model parameters. Section 2.2

reviews the general PMP approach, discusses various methods of parameter specification in this context, and points out a fundamental inconsistency of this methodology. A simple alternative calibration and estimation procedure based on first order necessary conditions is introduced in section 2.3. Subsequently, this alternative is illustrated with respect to various specifications of optimisation models. The first set of models in section 2.4 tries to rationalise non-linear objective functions and constraints in programming models as an implicit representation of a real economic phenomenon for which not enough direct data information exists. These models draw a little more heavily on the agricultural programming literature and remain close to the typical PMP application and related heuristic justifications. The second set of models in sections 2.5 and 2.6 encompass various optimisation models which have been used for econometric modelling of multi-output multi-input systems with fixed allocable factors or constitute extensions. They vary by technological assumptions (non-joint versus joint variable inputs), complexity (number of endogenous variable types), and behavioural assumptions (profit maximisation versus expected utility maximisation). The final section 2.7 summarises the major findings and draws the connection to the subsequent chapter of this study.

2.2 The Positive Mathematical Programming Approach (PMP)

2.2.1 The General Concept of Positive Mathematical Programming

This section gives an introduction to Positive Mathematical Programming (PMP) as it has been applied in many sectoral, regional and farm modelling activities in agriculture over the past 15 years. It provides the base for subsequent comparisons with alternative calibration and estimation approaches for programming models.

Consider the following profit maximising linear programming problem:

$$\begin{aligned} &\underset{\mathbf{x}}{\text{Max}}\; Z = \mathbf{p}'\mathbf{x} - \mathbf{c}'\mathbf{x} \\ &\text{subject to} \\ &\mathbf{A}\mathbf{x} \le \mathbf{b} \qquad [\lambda] \\ &\mathbf{x} \ge \mathbf{0} \end{aligned} \tag{2.1}$$

where

Z = objective function value

$\mathbf{p}$ = (N×1) vector of product prices

[2] SCHAIBLE (1997) and (2000) uses a very similar approach, but does not refer at all to the PMP literature.

$\mathbf{x}$ = (N×1) vector of production activity levels

$\mathbf{c}$ = (N×1) vector of accounting cost per unit of activity

$\mathbf{A}$ = (M×N) matrix of coefficients in resource constraints

$\mathbf{b}$ = (M×1) vector of available resource quantities

$\boldsymbol{\lambda}$ = (M×1) vector of dual variables associated with the resource constraints

The problem (2.1) might represent an optimisation problem at the farm or some aggregate level as commonly used in applied agricultural policy modelling. Given an appropriate data set based on farm accounting or sectoral (regional) averages, the solution of this problem will, in general, not reproduce the observed allocations of fixed resources to the production activities. In most cases overspecialisation of the solution occurs, because the number of empirically justified (or available) resource constraints is usually well below the number of observed activities. Since the number of nonzero activities in an LP framework is upper bounded by the number of resource constraints, overspecialisation must occur by design.

The problem of overspecialisation is generally more severe in aggregate models for several reasons (see also HOWITT 1995a: 330):

1. The number of empirically justified constraints relative to the number of observed production activities is smaller compared to the farm level.
2. Data, time and computational restrictions oftentimes do not allow specifying relevant non-linearity in aggregate technology that would force more production activities into the solution.
3. For the same reasons, output price endogeneity and risk behaviour, which would both imply some tendency towards diversification, are often not incorporated into the objective function of the model.

Model solutions that deviate substantially from observed production quantities are certainly not appealing in the context of 'selling' these models to political decision-makers. Neglecting so many relevant factors determining observed supply behaviour in fact does make the usefulness of these models for policy consulting purposes very questionable. Therefore, applied modellers invested significant effort into calibrating linear programming models to better reproduce observed base year quantities or averages over several years. Calibration within the framework of LP's was mainly done by introducing additional rotational constraints or simply by adding upper and lower bounds on certain production activities. Apart from the weak theoretical and

empirical justification of the additional constraints at the aggregate level, they also (inappropriately) constrain the set of possible simulation results such that ad-hoc mechanisms were incorporated to increase model flexibility in simulation runs (flexibility constraints). An extensive overview and discussion on 'pre-PMP' calibration approaches is given in HAZELL and NORTON (1986) and BAUER and KASNAKOGLU (1990).

The introduction of an objective function non-linear in variables to explicitly model risk behaviour or endogenous prices yields interior solutions for certain production activities - independent of the constraints - and thereby provides some relief from the overspecialisation problem. However, experiences show (e.g. MEISTER et al. 1978) that the problem does not fully disappear. Furthermore, even if all observed production activities are also nonzero in the optimal solution, deviations in optimal levels from observed levels will still occur and the additional application of calibration techniques with the above mentioned negative implications for the models simulation response is required.

With this background, the success of PMP in applied sector modelling activities is understandable, because it is not only an elegant procedure to *exactly* calibrate the model solution to observed quantities, but also promises to yield a more flexible and realistic simulation behaviour of the model.

The general idea is to use information contained in dual variables of calibration constraints, which bound the LP-problem to observed activity levels (Phase 1). These dual values are used to specify a non-linear objective function such that observed activity levels are reproduced by the optimal solution of the new programming problem without bounds (Phase 2).

Phase 1 of this procedure is formally described by extending model (2.1) in the following way:

$$
\begin{aligned}
&\max_{\mathbf{x}} Z = \mathbf{p}'\mathbf{x} - \mathbf{c}'\mathbf{x} \\
&\text{subject to} \\
&\mathbf{A}\mathbf{x} \le \mathbf{b} \qquad [\boldsymbol{\lambda}] \\
&\mathbf{x} \le (\mathbf{x}^{o} + \mathrm{e}) \qquad [\boldsymbol{\rho}] \\
&\mathbf{x} \ge [\mathbf{0}]
\end{aligned}
\tag{2.2}
$$

where

$\mathbf{x}^o$ = (N×1) vector of observed activity levels

$\boldsymbol{\varepsilon}$ = (N×1) vector of a small positive numbers

$\boldsymbol{\rho}$ = dual variables associated with the calibration constraints

The addition of the calibration constraints will force the optimal solution of the linear programming model (2.1) to exactly reproduce the observed base year activity levels $\mathbf{x}^o$, given that the specified resource constraints allow for this solution (which they should if the data are consistent, see HAZELL and NORTON 1986: 266f). 'Exactly' is accurately understood to mean within the range of the positive perturbations of the calibration constraints, $\boldsymbol{\varepsilon}$, which are included to guarantee that all binding resource constraints of model (2.1) remain binding here.

We can partition the vector $\mathbf{x}$ into two subsets, an ((N-M)×1) vector of 'preferable' activities, $\mathbf{x}^p$, which are constrained by the calibration constraints, and a (M×1) vector of 'marginal' activities, $\mathbf{x}^m$, which are constrained by the resource constraints. To simplify notation, without loss of generality, we assume that all elements in $\mathbf{x}^o$ are nonzero and all resource constraints are binding. Then, the Kuhn-Tucker conditions imply that

$$\boldsymbol{\rho}^p = \mathbf{p}^p - \mathbf{c}^p - \mathbf{A}^{p\prime}\boldsymbol{\lambda} \tag{2.3}$$

$$\boldsymbol{\rho}^m = [\mathbf{0}] \tag{2.4}$$

$$\boldsymbol{\lambda} = \left(\mathbf{A}^{m\prime}\right)^{-1}\left(\mathbf{p}^m - \mathbf{c}^m\right) \tag{2.5}$$

where the superscripts p and m indicate subsets of original vectors and matrices corresponding to preferable and marginal activities, respectively. The dual values of the calibration constraints are zero for marginal activities ($\boldsymbol{\rho}^m$) and equal to the difference of price and marginal cost for preferable activities ($\boldsymbol{\rho}^p$), latter being the sum of variable cost per activity unit ($\mathbf{c}$) and the marginal cost of using fixed resources ($\mathbf{A}^{p\prime}\boldsymbol{\lambda}$) . It should be noted here, that the dual values of the resource constraints ($\boldsymbol{\lambda}$) only depend on objective function entries and coefficients of marginal activities.[3]

In *Phase 2* of the procedure, the $\boldsymbol{\rho}^p$ are employed to specify a non-linear objective function such that the marginal cost of the preferable activities are equal to their respective prices at the base year activity levels $\mathbf{x}^o$. Given that the implied variable cost function has the right curvature properties (convex in activity levels) the solution to the resulting programming problem will be a

[3] The dual values will certainly be smaller than those that would be obtained without the calibration constraints in model (1), since marginal instead of preferable activities now determine the value of the resources. It is not exactly clear to the author in what context HOWITT (1995a, p.332) said that the addition of calibration constraints does not change the dual values associated with - what he calls - 'general constraints'. The author cannot follow HOWITT's 'proposition 3' in the appendix.

'boundary point, which is the combination of binding constraints and first order conditions' (HOWITT 1995a: 330).

In principle, any type of non-linear function with the required properties qualifies for this step. For reasons of computational simplicity and lacking strong arguments for other type of functions, a quadratic cost function is often employed (exceptions: PARIS and HOWITT 1998 and 2000). The general version of this variable cost function to be specified is then

$$C^v = \mathbf{d}'\mathbf{x} + \frac{1}{2}\mathbf{x}'\mathbf{Q}\mathbf{x} \tag{2.6}$$

with

d = (N×1) vector of parameters associated with the linear term and

Q = (N×N) symmetric, positive (semi-) definite matrix of parameters associated with the quadratic term.

The parameters are then specified such that the 'marginal variable cost' ($\mathbf{MC}^V$) fulfil

$$\mathbf{MC}^V = \frac{\partial C^v(\mathbf{x}^o)}{\partial \mathbf{x}} = \mathbf{d} + \mathbf{Q}\mathbf{x}^o = \mathbf{c} + \boldsymbol{\rho}\,. \tag{2.7}$$

Note, however, that the derivative of this *variable* cost function does not incorporate the opportunity cost of fixed resources ($\mathbf{A}^{p\prime}\boldsymbol{\lambda}$). Those remain captured in the ultimate model by the dual values of the resource constraints.

Given that we have a set of parameters satisfying (2.7) we obtain the final non-linear programming problem that reproduces observed activity levels as

$$\begin{aligned} &\max_{\mathbf{x}} Z = \mathbf{p}'\mathbf{x} - \mathbf{d}'\mathbf{x} - \frac{1}{2}\mathbf{x}'\mathbf{Q}\mathbf{x} \\ &\text{subject to} \\ &\mathbf{A}\mathbf{x} \le \mathbf{b} \qquad [\boldsymbol{\lambda}] \\ &\mathbf{x} \ge \mathbf{0} \end{aligned} \tag{2.8}$$

It should be noted at this point that the dual values of the resource constraints in model (2.8) at $\mathbf{x}^o$ do not differ from the one in model (2.2). They are still determined by the marginal profitability of the marginal activities at their observed levels $\mathbf{x}^{om}$, $(\mathbf{A}^{m\prime})^{-1}$ $[\mathbf{p}^m - (\mathbf{d}^m + \mathbf{x}^{om}\mathbf{q}^m)]$, which remains equal to $(\mathbf{A}^{m\prime})^{-1}[\mathbf{p}^m - \mathbf{c}^m]$ in the specification step, because of (2.4) and (2.7). Consequently, the value of equation (2.5) remains unchanged.

2.2.2 The Parameter Specification Problem

Calibration of an agricultural farm, regional or sectoral programming model to observed quantities is not the distinctive property of the PMP-approach. This can be achieved by appropriate constraints - see model (2.2) - as well. More interesting is, whether a PMP calibrated programming model is able to capture the behavioural response of farmers to changing economic conditions, so that it is capable of evaluating impacts of political-, market-, or technical developments on agriculture. With regard to this simulation behaviour, the methodology has two clear advantages compared to a linear programming model calibrated by constraints:

- the response is not restricted by weakly justified constraints
- the response is smooth compared to a linear programming problem.

However, these comparative advantages do not imply any quantitative realism with respect to the response behaviour of the PMP-calibrated model. In fact, the first and second order conditions - (2.7) and the positive (semi-) definiteness of **Q** - still allow for almost any magnitude of response behaviour of the resulting model.

The problem of condition (2.7) is that it implies an underdetermined specification problem as long as we consider a flexible functional form. In the case of the second order flexible quadratic function we have N+N(N+1)/2 parameters which we try to specify on the basis of 2N pieces of information (the marginal variable cost equations (2.7)). There are an infinite number of parameter sets which satisfy these conditions, i.e. lead to a perfectly calibrating model, but each set implies a different response behaviour to changing economic incentives.

In order to see this, we derive the supply functions implied by the PMP calibrated model (2.8). If we start from the Lagrangian formulation

$$\text{(2.9)} \qquad L(\mathbf{x}) = \mathbf{p}'\mathbf{x} - \mathbf{d}'\mathbf{x} - 0.5\mathbf{x}'\mathbf{Q}\mathbf{x} + \boldsymbol{\lambda}\left[\mathbf{b} - \mathbf{A}\mathbf{x}\right]$$

and continue to assume that all optimal activity levels are positive we obtain the first order conditions in gradient format as

$$\text{(2.10)} \qquad \frac{\partial \mathbf{L}}{\partial \mathbf{x}} = \mathbf{p} - \mathbf{d} - \mathbf{Q}\mathbf{x} - \mathbf{A}'\boldsymbol{\lambda} = \mathbf{0}$$

and

$$\text{(2.11)} \qquad \frac{\partial \mathbf{L}}{\partial \boldsymbol{\lambda}} = \mathbf{b} - \mathbf{A}\mathbf{x} = \mathbf{0}\,.$$

Solving (2.10) for **x** results in

$$\mathbf{x} = \mathbf{Q}^{-1}\left(\mathbf{p} - \mathbf{d} - \mathbf{A}'\boldsymbol{\lambda}\right) \tag{2.12}$$

and substituting the right hand side of (2.12) into (2.11) allows to solve for

$$\boldsymbol{\lambda} = \left(\mathbf{A}\mathbf{Q}^{-1}\mathbf{A}'\right)^{-1}\left(\mathbf{A}\mathbf{Q}^{-1}\left(\mathbf{p} - \mathbf{d}\right) - \mathbf{b}\right). \tag{2.13}$$

The vector of optimal activity levels as a function of exogenous model parameter can then be expressed as

$$\mathbf{x} = \mathbf{Q}^{-1}\left(\mathbf{p} - \mathbf{d}\right) - \mathbf{Q}^{-1}\mathbf{A}'\left(\mathbf{A}\mathbf{Q}^{-1}\mathbf{A}'\right)^{-1}\left(\mathbf{A}\mathbf{Q}^{-1}\left(\mathbf{p} - \mathbf{d}\right) - \mathbf{b}\right). \tag{2.14}$$

The gradient of (2.14) with respect to the price vector is proportional to the marginal supply response in this case (since product supply is constant per activity unit) and given by

$$\frac{\partial \mathbf{x}}{\partial \mathbf{p}} = \mathbf{Q}^{-1} - \mathbf{Q}^{-1}\mathbf{A}'\left(\mathbf{A}\mathbf{Q}^{-1}\mathbf{A}'\right)^{-1}\mathbf{A}\mathbf{Q}^{-1} \tag{2.15}$$

which finally reveals that the full **Q**-matrix is relevant for the supply response of each single product. This is even true when **Q** is diagonal (and consequently $\mathbf{Q}^{-1}$ as well), because the fixed allocable inputs (resource constraints) still link all production activities with each other. The second summand in (2.15) which is $-\mathbf{Q}^{-1}\mathbf{A}'$ times the gradient of $\boldsymbol{\lambda}$ with respect to **p** ensures that all elements of $\mathbf{Q}^{-1}$ enter each element of the supply gradient.

The different methods developed to choose among the infinite number of calibrating parameter sets increasingly recognised the need to introduce additional information in order to avoid an arbitrary simulation behaviour. We give here a short overview on the principles employed without an extensive discussion (see also UMSTÄTTER 1999: 30ff. or for a detailed evaluation RÖHM 2001:8ff. with respect to some of the approaches mentioned below):

An Early Specification Rule

In the – surprisingly long lasting – 'early' days of PMP the specification problem with respect to the quadratic cost function was simply solved by letting **d** = **c** and setting all off-diagonal elements of **Q** to 0 (e.g. HOWITT and MEAN 1983, BAUER and KASNAKOGLU 1990, SCHMITZ 1994; ARFINI and PARIS 1995). The N diagonal elements of **Q**, q_{ii}, were then calculated as

$$q_{ii} = \frac{\rho_i}{x_i^o} \text{ for all } i = 1,...,N. \tag{2.16}$$

It is easily verified that the resulting variable cost function satisfies condition (2.7). This specification rule leads to a cost function which is linear in 'marginal' activity levels, because the elements of $\rho^m = 0$. This in turn implies that λ remains constant, because it is determined by the profitability of the marginal activities alone which is constant per activity unit. Consequently, a price increase for products of the preferable production activities leads to a substitution of marginal activities, but leaves the other preferable activity levels unchanged until the first marginal activity is driven out of the basis.

This specification is purely motivated by computational simplicity in the absence of additional information. Its repeated use can only be explained by a focus on the calibration property in hope that a rich technological specification in terms of constraints would provide a realistic simulation response. In hindsight, it is easy to argue that technological constraints which are not even closely capable of reproducing base year observations are not in any way more likely to capture behavioural response to changing economic incentives.

Ex-post simulations performed by CYPRIS (2000) with the German regionalised sector model RAUMIS show that this approach results in a very poor response behaviour of the resulting model characterised by strong overreactions to changes in economic incentives (i.e. high implied elasticities).

PARIS (1988) used an alternative specification rule. They set the linear cost function parameters d = 0 in addition to the off diagonal elements of **Q**, and calculated

$$q_{ii} = \frac{c_i + \rho_i}{x_i^o} \quad \forall i = 1,...,N \ . \tag{2.17}$$

which achieves positive diagonal elements of **Q** also for the marginal activities. In this case, the reduction of marginal activities caused by the expansion of a preferable activity would immediately change the dual values of the resource constraints, π, which in turn alters the optimal solution for other preferable activities. Although this is a generally more realistic property of (aggregate) producer response, the quantitative specification remains just as arbitrary.

Average Cost Approach

If one is willing to assume that the observed vector of accounting cost per activity unit, **c**, is equal to the average cost of the crop specific variable cost function, one can satisfy condition (2.7) alternatively by letting

$$(2.18) \qquad q_{ii} = \frac{2\rho_i}{x_i^o} \text{ and } d_i = c_i - \rho_i \quad \forall i = 1,...,N .$$

In comparison with specification (2.16), the diagonal elements get larger implying a reduced price elasticity. The average cost approach excludes positive off-diagonal elements of **Q** by definition, because crop specific average cost is not defined in this case. In fact, the allocation of variable inputs to certain crop activities is not consistent with the joint technology underlying a multi-product cost function.

This approach has been used in HECKELEI and BRITZ (2000) in the context of an ex-post simulation exercise for comparative purposes (see also chapter 4). For further discussion see also GOHIN and CHANTREUIL 1999.

Use of Exogenous Supply Elasticities

A generally more convincing specification is the incorporation of exogenous elasticities. It reduces the role of PMP to all it really can be in the context of just one observation on activity levels: a calibration method.

Existing applications (for example HELMING et al. 2001) are restricted to the use of exogenous *own*-price elasticities $\bar{\varepsilon}_{ii}$. The off-diagonal elements of **Q** are set to zero and the marginal effect of price changes on the shadow prices λ is ignored (second summand in (2.14) vanishes). In this case, the partial derivative $\partial x_i / \partial p_i$ is equal to q_{ii}^{-1} so that the elasticity formula evaluated at observed quantities can be solved directly for q_{ii} to obtain

$$(2.19) \qquad \bar{\varepsilon}_{ii} = \frac{1}{q_{ii}} \frac{p_i^o}{x_i^o} \Leftrightarrow q_{ii} = \frac{1}{\varepsilon_{ii}} \frac{p_i^o}{x_i^o} \quad \forall i = 1,...,N$$

as the appropriate value for a given ε_{ii}. In order to satisfy the calibration condition (2.7) the linear parameters of the variable cost function are then determined as

$$(2.20) \qquad d_i = c_i + \rho_i - q_{ii} x_i^o \quad \forall i = 1,...,N .$$

Because of the ignored effect on shadow prices of limited resources, the actual elasticities of the resulting model will deviate from $\bar{\varepsilon}_{ii}$. The exact calibration to exogenous own-price elasticities is generally possible but cannot always be obtained as a closed form solution. In chapter 3 a numerical procedure is used to solve for the implied values of the diagonal elements of **Q** in an

equivalent model setting but 'without PMP', i.e. without use of dual values from calibration constraints.[4]

Calibration with Maximum Entropy Criterion

PARIS and HOWITT (1998) address the issue of the potentially arbitrary parameter specification in a different way. They suggest a 'Maximum Entropy' procedure to generalise and objectify the calibration phase. It also includes a full specification of the **Q**-matrix and the employment of this information theoretic criterion allows to solve the underdetermined specification problem. An introduction to the Generalised Maximum Entropy methodology is given in chapter 3 section 2. The specific PMP approach by PARIS and HOWITT is presented in the context of a related application by HECKELEI and BRITZ (2000) in section 4.3. At this point we just want to make the following remarks with respect to their approach:

- The employment of the Maximum Entropy criterion generally allows for the use of more than one observation on activity levels which is also acknowledged by the authors
- However, their own application relies only on one observation leaving the resulting simulation behaviour again a product of rather arbitrary methodological details
- The recovered cost function satisfies condition (2.7) which makes their approach subject to the general criticism following in the subsequent sections.

Despite the criticism hinted at with the last two points, this paper by PARIS and HOWITT basically initiated the research presented in this study. The innovative idea of using an econometric criterion for the calibration of programming models was the first step to bridge the traditional gap between econometric models based on behavioural functions (supply and input demand functions) and the use of synthetic programming models in agricultural supply analysis.

Specification Based on Decreasing Marginal Yields

All the PMP specifications mentioned above specified a non-linear *cost* function. By assumption, they attribute the marginal mis-specification of the original linear model to the input side of the production problem. It is probably obvious, however, that a misrepresentation of how revenue depends on activity levels would have the same effect. In fact, the dual values on the calibration

[4] The same section also shows a calibration approach which is based upon prior information on the full matrix of elasticities by directly employing the elasticity form of equation (2.15). However, the exact calibration to this elasticity matrix is not possible anymore, because we then have more restrictions than parameters. A criterion must be employed which determines parameter such that the resulting elasticity matrix is as 'close as possible' to the prior information.

constraints can just as well be explained by decreasing marginal yields with increasing activity levels, which is not reflected by the constant yield assumption of model (2.1). HOWITT (1995a) uses this interpretation and introduces non-linear terms into the objective function to reflect differences between marginal and observed average crop yields caused by changing land quality.

The theoretical as well as empirical validity of the pure cost function and the pure yield function approach is very questionable. Any reasonable technological and behavioural assumption in agricultural production would make it highly unlikely that input application is changed but yield remains constant or vice versa. RÖHM (2001:51ff.) acknowledges this by combining the decreasing yield with the increasing cost assumption. However, his approach is also not based on a clear technological hypothesis, i.e. a well represented relationship between inputs and outputs, and again does not provide a strong empirical base for the specification of parameters and the implied simulation behaviour of the resulting model.

Therefore, we refrain from elaborating on the details of the yield function approach or combined yield/cost approaches. Instead we focus now on the general problem of the PMP-approach related to the 'phase 1' which would render its use for parameter estimation based on multiple observations problematic. Later, we move to a synoptic view of input use and output generation using explicit technological and behavioural assumptions.

2.2.3 Fundamental Inconsistency of PMP

The last section pointed out the danger of specifying models based on PMP that imply an arbitrary simulation behaviour. One problem is the thin information base provided by just one year of observations on activity levels. In fact, the data in this case do not provide any information on second order properties (Hessian matrix) of the objective function. If a change in economic incentives and the resulting behaviour is not observed, then the information for parameter specification must come from other sources. Even if one would be able to specify the 'true' model with respect to behavioural assumptions and functional form, the parameters are still not identified. The only convincing use of PMP with just one observation is the use as a calibration method in combination with elasticities or other exogenous information on technology or behavioural response with respect to changes in activity levels.

The main focus of this study, however, shall be the inclusion of additional data looking for the bridge to typical econometric models. PARIS and HOWITT (1998) already addressed the problem of introducing more than one observation. Chapters 3 and 4 will show that their suggestion to use Maximum Entropy techniques provide a feasible and flexible tool for this purpose. However, the

question we need to address first is, whether the PMP procedure itself is designed to make best use of additional data information. We show that the marginal conditions derived from the first phase of PMP are inappropriate. They represent a mis-specified model in the sense that the inclusion of additional observations will never allow to recover the underlying model which is assumed to have generated the data.

In order to see this, we will use some of the elements already introduced in the previous sections, but look at the methodology from an econometrician's point of view. This includes the assumption that the ultimate model to be specified is the 'true' model structure, or at least one that is believed to be a good approximation of the true model: Apparently, many PMP modellers thought that the final model with a non-linear objective function to be optimised under linear resource constraints is a reasonable representation of the behaviour of agricultural crop producers, otherwise it would not have made any sense to use this structure as the ultimate specification. The PMP procedure, however, enforces shadow prices and marginal cost values that differ from the ones implied by the non-linear model.

Suppose the quadratic model (2.8) is the true data generating process. The derivations (2.9) to (2.13) have shown that the shadow prices of the resource constraints under the assumption that all activity levels are positive at the optimum can be calculated as $\boldsymbol{\lambda} = \left(\mathbf{A}\mathbf{Q}^{-1}\mathbf{A}'\right)^{-1}\left(\mathbf{A}\mathbf{Q}^{-1}\left(\mathbf{p}-\mathbf{d}\right)-\mathbf{b}\right)$. This is clearly different from the dual values of the resource constraints obtained in the first phase of PMP (see equation (2.5)) which only depended on quantities related to the marginal activities (superscript 'm') and were given by $\boldsymbol{\lambda} = \left(\mathbf{A}^{\mathrm{m}}{}'\right)^{-1}\left(\mathbf{p}^{\mathrm{m}}-\mathbf{c}^{\mathrm{m}}\right)$. The second phase of PMP then uses these wrong dual values at the observed activity levels through enforcement of the 'marginal cost' equations (2.7), thereby implicitly imposing wrong values for the marginal variable cost as well. Given this discrepancy, it is impossible to recover the true non-linear objective function no matter how many observations on activity levels are used. The use of the *biased* marginal cost equations as estimating equations in some econometric exercise with multiple observations generally leads to inconsistent estimates. The PMP approach is fundamentally flawed in the sense that it imposes first order conditions which are incompatible with the non-linear model it ultimately tries to recover.

In principle, the problems with the dual values of the resource constraints have been recognised before. GOHIN and CHANTREUIL (1999), CYPRIS (2000) and RÖHM (2001) suggest to include observed land rents into the specification step to ensure a more reasonable value. This can either

be done by introducing a land rent activity at this price into the original linear model or by adjusting the conditions for specification. WILD (2000) employs simulation exercises to show the impossibility to recover a true quadratic model with more than one observation based on phase 1 of PMP. He also shows an alternative calibration approach specifically designed for the quadratic specification which does recover the true model by simultaneously calibrating parameters and shadow price of land.

GOHIN and CHANTREUIL and WILD already observe that in the case of a simple model structure with just one resource constraint, phase 1 of PMP is not needed as the implied value of the shadow price of land can be deduced directly. Below we introduce a general alternative which does not require phase 1 of PMP to calibrate or estimate any programming model even for more complex constraint structures. A direct use of the first order conditions of the assumed behavioural optimisation model makes the use of distorted shadow prices and thereby the use of the PMP-approach altogether obsolete.

2.2.4 Summary on Review of PMP

Before leaving the PMP approach we want to close this section with a short summary of the merits and problems associated with the main body of PMP applications in the context of agricultural sector models.

(1) The PMP approach provides an elegant way to calibrate programming models to observed behaviour and renders a more realistic smooth aggregate supply response relative to a linear programming model. These merits lead to a widespread application of PMP approaches in the context of aggregate agricultural programming models.

(2) The dual values associated with the calibration constraints in phase 1 of PMP potentially capture any type of marginal model mis-specification of technology, data errors, aggregation bias, representation of risk behaviour, price expectations, non-linear constraints etc. For an intelligent model specification and appropriate interpretation of results, explicit assumptions used in the model specification are desirable. Calibration and estimation procedures for these flexible model specifications should be available.

(3) One observation on base year allocations alone does not contain any information on how the marginal incentives change if one moves away from the observed allocation. The infinite number of calibrating sets of parameters generally imply a different simulation response of the calibrated model. Extremely unreasonable supply responses have been generated in the

past with oversimplified PMP specifications. For a sensible specification, other information has to be employed, for example based on given supply elasticities.

(4) In addition or alternatively, the incorporation of more than one observation which – in combination with structural assumptions on the objective function and constraints – would enable to *estimate* the model parameters underlying the observed response behaviour of producers. The literature already provides a generally applicable tool in form of the Maximum Entropy criterion to accommodate any number of observations for this purpose, but in the context of positive degrees of freedom other statistical criteria will work as well.

(5) The shadow prices of resource constraints and the marginal variable costs at observed quantities enforced by the PMP approach are generally incompatible with the marginal conditions of the non-linear model to be specified. A consistent econometric approach that allows to recover the specification of the true model with increasing data information must consequently employ other estimating equations than the ones used in the PMP calibration approach which have to be unbiased under the assumed economic model.

2.3 A General Alternative to PMP

The 'general' alternative to PMP with respect to calibrating or estimating a programming model is nothing but a simple methodological principle: always to directly use the first order conditions of the very optimisation model that is assumed to represent or approximate producer behaviour and is suitable for the simulation needs of the analysts. No first phase calculating dual values of calibration constraints based on a different model is necessary. We can avoid the implied methodological inconsistency altogether and generally estimate shadow prices of resource constraints simultaneously with the other parameters of the model.

The basic principle can be illustrated by writing a general programming model with an objective function $h(\mathbf{y}|\boldsymbol{\alpha})$ to be optimised subject to a constraint vector $\mathbf{g}(\mathbf{y}|\boldsymbol{\beta}) = 0$ in Lagrangian form:

$$L(\mathbf{y},\boldsymbol{\lambda}|\boldsymbol{\alpha},\boldsymbol{\beta}) = h(\mathbf{y}|\boldsymbol{\alpha}) + \boldsymbol{\lambda}'[\mathbf{g}(\mathbf{y}|\boldsymbol{\beta})], \tag{2.21}$$

where $\mathbf{y}$, $\boldsymbol{\lambda}$, $\boldsymbol{\alpha}$, and $\boldsymbol{\beta}$ represent column vectors of endogenous variables, unknown dual values, parameters of the objective function, and parameters of the constraints, respectively. The appropriate first order optimality conditions are the gradients with respect to $\mathbf{y}$ and $\boldsymbol{\lambda}$ set to zero:

$$\frac{\partial L}{\partial \mathbf{y}} = \frac{\partial h(\mathbf{y}|\boldsymbol{\alpha})}{\partial \mathbf{y}} + \boldsymbol{\lambda}' \frac{\partial \mathbf{g}(\mathbf{y}|\boldsymbol{\beta})}{\partial \mathbf{y}} = \mathbf{0} \tag{2.22}$$

$$\frac{\partial L}{\partial \boldsymbol{\lambda}} = \mathbf{g}(\mathbf{y}\,|\,\boldsymbol{\beta}) = \mathbf{0}\,. \tag{2.23}$$

For the case of inequality constraints $\mathbf{g}(\mathbf{y}|\boldsymbol{\beta}) \leq 0$ we need to substitute the gradient with respect to $\boldsymbol{\lambda}$ by the complementary slackness representation[5]

$$\frac{\partial L}{\partial \boldsymbol{\lambda}} = \mathbf{g}(\mathbf{y}\,|\,\boldsymbol{\beta}) \leq \mathbf{0};\quad \boldsymbol{\lambda} \odot \mathbf{g}(\mathbf{y}\,|\,\boldsymbol{\beta}) = \mathbf{0} \tag{2.24}$$

The unknowns $\boldsymbol{\lambda}$, $\boldsymbol{\alpha}$, and $\boldsymbol{\beta}$ of these Kuhn Tucker conditions can be estimated with some econometric criterion directly applied to these equations. Depending on the parametric specification appropriate curvature restrictions (second order conditions) might have to be enforced as well.

The direct use of optimality conditions for estimation is certainly not new by itself. In the context of investment models, for example, the dynamic equivalents of Kuhn Tucker conditions, the Euler equations, have been frequently used as estimating equations to overcome analytical and empirical problems for more complex models (CHIRINKO 1993:1893f). However, their employment as an alternative to PMP or to the estimation of behavioural functions in the context of multi-output agricultural supply models has not been considered. One of the examples in the subsequent section will show that this approach is not only useful for the estimation of typical agricultural programming models but also provides a flexible alternative for estimating parameters of duality based behavioural functions with explicit allocation of fixed factors. In this context, the only difference left between programming and econometric models is the model form used for simulation purposes.

In hindsight it might be difficult to understand why this principle had not been applied earlier for the purpose of obtaining a programming model based on observed behaviour. One reason might be the often complex structure of inequality constraints employed in agricultural programming models which seemed to make the approach infeasible for larger models. However, in most PMP publications the calibrated models have very simple structures and often only binding constraints so that the complete set of first order conditions is very limited. Also, the feasibility in more complex situations is far from proven and chapter 3 of this study will start to investigate this question. A more likely explanation is that most programmers did not see the need nor the

[5] The symbol '$\odot$' represents the Hadamard or element-wise product of two matrices. If a_{ij} and b_{ij} are the elements of two matrices with equal dimension, **A** and **B**, then $\mathbf{A} \odot \mathbf{B} = \mathbf{C}$, where **C** is of the same dimension as **A**, **B** and each element of **C** is defined as $c_{ij} = a_{ij} \cdot b_{ij}$.

possibility to include more than one observation and to actually estimate their programming models. At the same time econometricians did not look at this body of literature as a potential alternative to analyse producer behaviour under maintained optimising assumptions. Therefore, nobody seemed to have asked the question of consistency, i.e. is it possible to recover more and more closely the true model if the data information is increased. One would have quickly come to realise that the PMP paradigm of using dual values from calibration constraints of a different model than the one to be recovered is neither useful nor necessary.

The most innovative PMP-proponents PARIS and HOWITT (2001) and PARIS (2001) came – in some sense – very close to the alternative principle of calibration in a recent conference contribution and journal article. Their 'Symmetric Positive Equilibrium Problem' (SPEP) calibrates a multi-input multi-output model based on marginal cost constraints. The following differences to previous PMP applications apply: (1) They use a 'total' cost function which additionally includes the prices of 'limiting inputs' such as land, thereby rendering a final model specification with variable inputs only.[6] (2) The first phase with calibration constraints allows for land to be rented out if internal profitability is originally lower than an exogenous land rental price. (3) Their ultimate model specification is not an optimisation model but a 'primal-dual' equilibrium problem.

Apart from problematic methodological issues already prevailing in PARIS and HOWITT (1998) and discussed in section 4.3, there are structural shortcomings corresponding to these characteristics. (1) A clear inconsistency between the first phase model where at most the available quantity of land can be used by each farm and the final specification that is not bound by resource capacities. (2) A hardly defendable specification of the 'first phase' that allows farms to *rent out* land but not to *lease* land. (3) An at least 'veiled' interpretation of the final model specification with respect to the behavioural assumptions.[7]

Another way to look at the 'SPEP' approach is to see that the implicit determination of the shadow prices of resources through the first phase of PMP (equation (2.5)) is at least lower bounded by exogenous input prices. But what can we do, if those prices are not available or if we do not want to apply a model that assumes variability for all inputs? The use of a land constraint in agricultural supply simulation models, for example, can be very appropriate and useful with

[6] The use of the term 'limiting inputs' is rather unfortunate from our point of view, because they are not limiting at all in the final model specification. Therefore, the stubborn use of quotes for this expression here.

respect to the reasonableness of model results. Other resources such as water for irrigation might be truly limiting as well. Also, how do we deal with policy restrictions that were already applied in the data period and for which shadow prices are not known?

In these empirically very relevant cases we have the possibility to apply the proposed estimation principle by using the full set of first order necessary conditions (2.22) and (2.23) or (2.22) and (2.24) as estimating (or calibrating) equations which include the constraints and the corresponding shadow prices. For the typical PMP model with a cost function quadratic in land allocations, those optimality conditions were already given by equations (2.10) and (2.11). An easy way out of this theoretical section would now be to summarise the findings and demonstrate functionality of the estimation approach and empirical relevance for this limited model in subsequent sections of the thesis. However, we want to go beyond the typical PMP dominated section of the literature in two respects:

(1) One conceptual shortcoming of resulting non-linear models from typical PMP applications is the limited rationalisation of the non-linear terms in the objective function. They are generally not founded on explicit behavioural or technological assumptions, but rather motivated with heuristic arguments related to 'other' determinants of supply behaviour not captured by the explicit part of the objective function and the constraints. Therefore, the next section takes a deeper look into possible rationalisations and starts with investigating the consequences of underlying heterogeneous land quality as one of the most prominent arguments. Then an interesting connection is drawn to a competing approach of calibrating programming models: The convex combination constraints advocated by McCarl (1982) and Önal and McCarl (1989 and 1991) to mitigate aggregation errors. The resulting model specification can also be interpreted as to represent a set of unknown resource constraints.

(2) The remaining sections look at various constrained optimisation models with non-linear technologies relevant in agricultural supply analysis. The main purpose is the derivation of first order conditions which could be used for estimating the parameters of these programming models. The models considered are not generally new but mostly drawn from econometric literature. However, they have not been used to specify explicit optimisation models based on observed behaviour. They shall illustrate the potential of the general approach which allows to combine the stability and flexibility of optimisation models in

[7] Paris (2001) explicitly states that the final model does not require an explicit profit maximisation or cost minimisation hypothesis and considers this an advantage of the approach (p.1060). But he does not give any

simulation analysis with the data based specification of econometric models. Furthermore, it opens the door for estimating more complex theoretical models with more choices for functional forms by not requiring closed form solutions to first order conditions in terms of the decision variables.

2.4 Rationalisation of Non-Linear Model Parts under Leontief Technologies

The content of this section is very close to the programming and PMP literature. It is assumed throughout that constant returns to scale or a technology of the Leontief type renders constant objective function contributions of activities per ha independent of the land allocated to the activity. The non-linear model parts implicitly represent some economic phenomenon for which no explicit formulation can be used due to data deficiencies.

2.4.1 Land Heterogeneity

The quality of land in terms of agricultural productivity differs already at farm level and even more at an aggregate regional level. Missing representation of different soil qualities in aggregate optimisation models is a potentially important source of error in explaining optimal allocation of land to crops. In the PMP context, HOWITT (1995a) attributes deviations of the linear programming solution from observed firm behaviour to the missing representation of heterogeneous land. He assumes decreasing marginal yields with increasing activity levels and uses available yield functions to calibrate the programming problem based on the dual values of the calibration constraints (p.333).

Apart from the problems of interpreting the dual values described earlier, this approach is rather restrictive as it assumes that the next ha allocated to a certain crop is always the most productive of all available ones with respect to this crop. In reality, however, a multitude of crops compete with each other on each type of land. Consequently, in a profit maximisation framework, allocation of each type of land is driven by relative profitability between crops on this land and not simply by the productivity of a specific crop compared to its profitability on other types of land. An expansion of a crop due to changing economic conditions can lead to higher, equal, or lower average yield and corresponding input applications depending on the specific substitutions on the various types of land which in turn depend on prices and technology.

insight on how behaviour is to be interpreted then.

Conceptually, the explicit incorporation of heterogeneous land in programming models is straightforward: The total available land restriction can be differentiated into different land qualities and crop activities defined for each of them. The solution of the optimisation problem gives the optimal allocation of land for each quality to the respective activities. In the context of aggregate supply models, however, the required data for this approach is rarely available. Neglecting the heterogeneity of land by simply using average activity definitions and a total land constraint is in fact a special case of aggregation error at which we will look later. Since variations in land quality has been one of the most prominent ad-hoc motivations for a non-linear objective functions in the context of PMP (HOWITT 1995a:333, HOWITT 1995b:152) we want to explicitly look at the implications of heterogeneous land specifications for the structure of the optimisation model.

Crop Specific Supply Functions for Land

In the context of Computable General Equilibrium (CGE) models, calibration of multi-crop models is often achieved by introducing different land supply functions for each crop (BURNIAUX et al. 1990; ABLER and SHORTLE 1992; for references on other approaches see GOHIN et al. 1999:83). Denoting the price of land for crop i as μ_i, the inverse supply functions can be expressed as

$$\mu_i = \mu_i(l_i) \quad \forall i = 1,...,N \tag{2.25}$$

with $\partial\mu_i/\partial l_i > 0$. The price of land for crop i could be interpreted as the marginal opportunity cost arising from substituting other crops. It is increasing in land allocated to this crop if one assumes that increased land allocation substitutes sequentially more profitable alternative uses of other qualities of land. Note that we have stepwise increasing opportunity cost in the context of a classical farm LP with homogeneous land as well. This, however, is caused by the combination of all constraints in the model. Here, the increasing opportunity cost are caused by different crop productivities on different qualities of land.

If we assume that (2.25) has a linear functional form, a representation of the land costs in the objective function would actually result in a quadratic objective function (see also HAZELL and NORTON 1986:202). To see this, let's subtract the land cost from linear profit in the context of a LP to get

$$
\begin{aligned}
Z &= \sum_{i=1}^{N} gm_i l_i - \mu_i l_i = \sum_{i=1}^{N} gm_i l_i - \left(a_i + b_i l_i\right) l_i \\
&= \sum_{i=1}^{N} \left(gm_i - a_i\right) l_i - b_i l_i^2
\end{aligned} \tag{2.26}
$$

where $gm_i = p_i - c_i$ is the gross margin of crop i and l_i the land allocated to crop i. Consequently, the original PMP-specification based on an objective function with only diagonal quadratic terms is perfectly consistent with linear crop specific land supply functions. HOWITT's claim that this approach is not appropriate for disaggregated models of agricultural supply might be true (1995b:153), but his PMP approach shows no conceptual difference.

The major problem of this model is that the opportunity cost of land allocated to a certain crop are typically directly related to the land constraint and the marginal profits of the competing crops within the overall optimisation problem. The ad-hoc introduction of endogenous land prices into the objective function without any connection to the other parts of the model constitutes a clear inconsistency.

Therefore, we leave the notion of crop specific land supply functions and have an explicit look at heterogeneous land in the context of typical linear programming problems.

Heterogeneous Land Represented by Land Classes

A more general representation of heterogeneous land can be achieved by distinguishing different land classes and corresponding Leontief technologies if the appropriate data were available. Suppose there are p = 1,...,P different qualities of land with availability L_p. For each land class there exist an optimal (N×1) vector of gross margin $\mathbf{gm}_p$ with each element representing the difference between revenue and variable cost per ha of the specific crop on land quality p. Denoting the vector of activity levels associated with the land class p as $\mathbf{l}_p$ and ignoring other technological constraints we can write the overall gross margin maximisation problem as

$$
\begin{aligned}
&\max_{\mathbf{l}_p} Z = \sum_{p=1}^{P} \mathbf{gm}_p' \mathbf{l}_p \\
&\text{subject to} \\
&\mathbf{u}' \mathbf{l}_p \le L_p \qquad \left[\lambda_p\right] \quad \forall p = 1,...,P \\
&\mathbf{l}_p \ge \mathbf{0} \quad \forall p = 1,...,P
\end{aligned} \tag{2.27}
$$

The solution to (2.27) exist of boundary points in the sense that each class of land p is allocated entirely to the crop with the highest gross margin per ha on this land quality. The shadow prices λ_p of each land constraint equals the gross margin of this crop. A model of this kind will

generally result in less specialised production programs compared to equivalent LP-models without differentiation of the land quality. The implied crop supply functions increase stepwise with increasing product prices, each step representing an additional quality of land claimed by the respective crop. The more land classes are distinguished, the more smooth the crop supply function becomes.

Contrary to the model considered above with increasing supply functions for land, the marginal cost of expanding a crop's production quantity here is now explicitly linked to the next best use on each land quality. Although the (opportunity) cost of land rises with expanding activity levels in both cases, the average gross margin of a crop per unit of total land allocated is not constant anymore in (2.27). Whether it increases or decreases with higher activity levels depends on the specific profitability structure of all crops on the different land qualities.[8]

The model's realism could be further enhanced by defining suitable crop rotations instead of crop specific activities on each land quality which would better mirror typical farmer behaviour. The implications for crop supply response and opportunity cost of land would not be changed. The major problem of empirically specifying model (2.27) is, however, the often limited or costly information on land quality distribution for the coverage of the model as well as the extensive work necessary to define the production activities on the different qualities of land. This is the main reason for the often assumed homogeneity of land, since data on total agricultural area and average gross margins with related input and output quantities is comparatively easy to get.

The question is, in what sense additional non-linear terms in the models objective function could implicitly capture behaviour resulting from neglected heterogeneous land qualities. Could it be seen as an 'approximating device' whose parameters can be estimated based on observed aggregate behaviour and thereby substitute for costly data and model specification work? At this point we only know that the general effect of non-linear objective function terms goes into the same direction: it reduces overspecialisation and provides a more smooth supply response. But how far does the approximation of the differentiated model carry? What type of error prone simplifications are incorporated that could potentially harm the quality of the model results? In order to address these questions we now turn to the underlying more general issue of aggregation in linear programming models.

[8] Heterogeneous land quality is not only relevant for the substitution of crops but also with respect to participation in policy programs (see, for example, RYGNESTAD and FRASER 1996 and FRASER 1997)

2.4.2 Aggregation: Convex Combination Constraints Revisited

Neglecting heterogeneous land in linear programming models is in fact a special case of the more general aggregation problem of linear programs that has attracted considerable attention in the past. The focus of this literature has been the aggregation of farm programs to some regional or sectoral aggregate programming model. DAY (1963) developed conditions for exact aggregation by posing the following problem:[9] Suppose there exist P firm models with numbers of activities and constraints of identical size where the model of the p^{th} firm can be represented as[10]

$$\begin{aligned} &\max_{\mathbf{x}_p} Z = \mathbf{gm}_p{}' \mathbf{x}_p \\ &\text{subject to} \\ &\mathbf{A}_p{}' \mathbf{x}_p \le \mathbf{b}_p \\ &\mathbf{x}_p \ge \mathbf{0} \end{aligned} \qquad (2.28)$$

Under what conditions can we find an aggregate program with the same size of the form

$$\begin{aligned} &\max_{\mathbf{x}} Z = \mathbf{gm}' \mathbf{x} \\ &\text{subject to} \\ &\mathbf{A}' \mathbf{x} \le \mathbf{b} \\ &\mathbf{x} \ge \mathbf{0} \end{aligned} \qquad (2.29)$$

such that $\mathbf{x} = \sum_p \mathbf{x}^p$. DAY concludes that exact aggregation is only possible if the firms are technologically homogeneous with $\mathbf{A} = \mathbf{A}_p$ for all firms and have proportional vectors of objective function entries and resource capacities ($\mathbf{gm} = \alpha\mathbf{gm}_p$ and $\mathbf{b} = \beta\mathbf{b}_p$ with α and β being positive scalars). If the conditions are not met, the aggregate model implies unrealistic free movement of resources between uses causing overspecialisation of the aggregate model. These aggregation conditions are very restrictive and subsequent attempts of generalisation (MILLER 1966; LEE 1966; PARIS and RAUSSER 1973) did not greatly improve upon these with respect to empirical consequences, although PARIS (1980) at least relaxed the identical size restriction with respect to the aggregate model. To this date, the work by DAY serves as an argument to distinguish farm types or regions within aggregate models to come closer to the homogeneity condition and reduce the aggregation error.

[9] The presentation of the aggregation problem by DAY (1963) is based on ÖNAL and MCCARL (1991a).
[10] Symbols are defined as in (2.1)

An empirically relevant approach to address the issue of exact aggregation has been developed by MCCARL (1982) and ÖNAL and MCCARL (1989 and 1991) based upon an extreme point representation of a linear model with full representation of each individual farm (MCCARL and SPREEN 1980). Assuming equal gross margins per unit of activity for each farm[11] this full model can be written as[12]

$$
\begin{aligned}
&\max_{\mathbf{x}_p} Z = \sum_{p=1}^{P} \mathbf{gm}'\mathbf{x}_p \\
&\text{subject to} \\
&\begin{bmatrix} \mathbf{A}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{A}_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \mathbf{A}_P \end{bmatrix}' \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_P \end{bmatrix} \leq \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_P \end{bmatrix} \qquad (2.30) \\
&\mathbf{x}_p \geq \mathbf{0} \quad \forall p = 1,...,P
\end{aligned}
$$

Note that – apart from the constant gross margins – the simple land allocation model with heterogeneous land quality (2.27) is a special case of (2.30). Building upon DANTZIG and WOLFE (1961), ÖNAL and MCCARL show that any feasible solution $(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_P)$ of the bounded set defined by the resource constraints in (2.30) can be expressed as a convex combination of all i = 1,...,E extreme points $(\hat{\mathbf{x}}_{1i}, \hat{\mathbf{x}}_{2i}, ..., \hat{\mathbf{x}}_{Pi})$ such that $\mathbf{x}_p = \sum_i \eta_i \hat{\mathbf{x}}_{pi}$ where $\eta_i \geq 0$ and $\sum_i \eta_i = 1$. It follows that $\mathbf{x} = \sum_p \mathbf{x}_p = \sum_p \sum_i \eta_i \hat{\mathbf{x}}_{pi} = \sum_i \eta_i \hat{\mathbf{x}}_i$ with $\hat{\mathbf{x}}_i = \sum_p \hat{\mathbf{x}}_{pi}$. We are now able to rewrite the model (2.30) in terms of convex combination constraints of aggregate quantities:

$$
\begin{aligned}
&\max_{\mathbf{x}_p} Z = \sum_{p=1}^{P} \mathbf{gm}'\mathbf{x}_p = \mathbf{gm}'\mathbf{x} \\
&\text{subject to} \\
&\mathbf{x} - \sum_{i=1}^{E} \eta_i \hat{\mathbf{x}}_i = 0 \qquad (2.31) \\
&\sum_{i=1}^{E} \eta_i = 1 \\
&\mathbf{x} \geq \mathbf{0}; \quad \eta_i \geq \mathbf{0} \; \forall i = 1,...,E
\end{aligned}
$$

[11] For the subsequent derivations it is important to have the same objective function entries for each farm. In order to keep notation in terms of 'activity levels' **x** we assume constant gross margins. The exposition could easily be generalised by including output and input quantities in the objective function multiplied by the respective prices and forcing different input-output combinations for each farm in the technological constraint set.

[12] ÖNAL and MCCARL (1991) actually develop their approach based upon a sector model with endogenous prices, but here we stay within the framework of supply models.

The important point of model (2.31) is that it exactly represents the disaggregated model (2.30) without using the set of constraints for each farm. Instead, the information on the aggregate technology is exclusively represented by aggregate extreme points $\hat{\mathbf{x}}_i$. ÖNAL and MCCARL suggest two approaches to obtain these extreme points for the empirical specification of the model: (1) To simulate different price scenarios with known farm models and aggregate the optimal quantities; (2) To use historically observed aggregate data assuming that those represent aggregate outcomes of firm level maximisation. Approach (1) seems empirically less relevant because the known farm models could be used directly for simulation purposes. However, it allows to demonstrate the functionality of the aggregation procedure (ÖNAL and MCCARL 1991:328ff.) and provides extreme points not restricted to historically observed economic conditions. Approach (2) opens the door to specify well behaved aggregate models without knowing the structure of technological conditions at farm level and is closely related to the idea of *estimating* a programming model using data on endogenous variables.

How do we now draw a connection to non-linear objective functions for programming models? The first step is to observe that the constraints $\mathbf{x} - \sum_i \eta_i \hat{\mathbf{x}}_i = 0$ and $\sum_i \eta_i = 1$ force the elements of the vector $\mathbf{x}$ to be on a convex hull around the extreme points $\hat{\mathbf{x}}_i$. Relevant with respect to the overall optimisation problem is, however, only the *frontier* of this convex hull, i.e. the set of vectors $\mathbf{x}^F$ for which no other vector $\tilde{\mathbf{x}}$ satisfying $\tilde{\mathbf{x}} - \sum_i \eta_i \hat{\mathbf{x}}_i = 0$ exists such that $\tilde{\mathbf{x}} \geq \mathbf{x}^F$. For a sufficient density of extreme points $\hat{\mathbf{x}}_i$, i.e. under the generally realistic assumption that a large number of farms with differentiated constraint sets exist, the frontier can be approximated by a convex non-linear function of aggregate activity levels $g(\mathbf{x}) = 0$. Substituting this function for the convex combination constraints into model (2.31) we get

$$\begin{aligned} &\max_{\mathbf{x}_p} Z = \mathbf{gm}'\mathbf{x} \\ &\text{subject to} \\ &g(\mathbf{x}) = 0 \\ &\mathbf{x} \geq \mathbf{0} \end{aligned} \qquad (2.32)$$

The Lagrangian formulation of (2.32) assuming positive optimal quantities for all elements of $\mathbf{x}$ is given by

$$L = \mathbf{gm}'\mathbf{x} + \gamma\big(g(\mathbf{x})\big) \qquad (2.33)$$

implying the first order necessary conditions

(2.34) $$\frac{\partial \mathrm{L}}{\partial \mathbf{x}} = \mathbf{gm} + \gamma\left(\frac{\partial \mathbf{g}(\mathbf{x})}{\partial \mathbf{x}}\right) = \mathbf{0}$$

(2.35) $$\frac{\partial \mathrm{L}}{\partial \gamma} = \mathrm{g}(\mathbf{x}) = 0\,.$$

Suppose we parameterise the constraint $\mathrm{g}(\mathbf{x}) = 0$ as a quadratic function of the form

(2.36) $$\mathrm{g}(\mathbf{x}) = \mathrm{a} - \mathbf{d}'\mathbf{x} - 0.5\mathbf{x}'\mathbf{Q}\mathbf{x} = 0\,.$$

Then condition (2.34) reads

(2.37) $$\frac{\partial \mathrm{L}}{\partial \mathbf{x}} = \mathbf{gm} + \gamma\left(-\mathbf{d} - \mathbf{Q}\mathbf{x}\right) = \mathbf{0}$$

In principle, equations (2.36) and (2.37) can serve as calibrating conditions or – after adding error terms to activity levels – as estimating equations for the aggregate programming model. However, when *calibrating* the model to one base year observation, the Lagrange multiplier γ is not identified and consequently can be set to one. Also noting that condition (2.36) is fulfilled by appropriately choosing the parameter 'a' for any given value of **d** and **Q**, the relevant condition for calibrating the parameters **d** and **Q** is then given by

(2.38) $$\mathbf{gm} - \mathbf{d} - \mathbf{Q}\mathbf{x} = \mathbf{0}$$

Now it is time to recall from above the corresponding first order condition of the typical PMP model with a cost function quadratic in activity levels:

(2.10) $$\frac{\partial \mathrm{L}}{\partial \mathbf{x}} = \mathbf{p} - \mathbf{d} - \mathbf{Q}\mathbf{x} - \mathbf{A}'\boldsymbol{\lambda} = \mathbf{0}$$

The difference to (2.38) is twofold: (1) the vector of prices **p** substitutes for the vector of gross margins **gm**; (2) an additional term $\mathbf{A}'\lambda$ accounts for the opportunity costs of fixed resources. Difference (1) simply implies a reparameterisation of the linear parameter **d** which does not contain the variable cost proportional to the activity level in (2.38) but does so in (2.10). Difference (2) would go away if one interpreted the linear resource constraints as valid at the aggregate level and included those in addition to the non-linear constraint in our 'frontier' model (2.32). Alternatively, equivalence of the two approaches is also obtained if one assumes that no valid resource constraints at the aggregate level exists for both models.

The 'equivalence', however, is a very limited one. It only implies that if certain values of **d** and **Q** calibrate one model to observed base year activity levels they also calibrate the other model to

the same observations. But the resulting programming models differ in structure, and consequently, simulation response for same values of **d** and **Q**. This is easy to see if one considers the non-linear constraint (2.36) of the frontier model. It forces the sum **d'x** + **x'Qx** to be equal to 'a' under all economic conditions. This is not the case for the model with the non-linear cost function. In the calibration practice, this difference is not very important since both models can be calibrated to the same set of exogenous elasticities (implying different **d**'s and **Q**'s). The only advantage of the frontier model at this point is the more explicit rationalisation of the model structure.

Once we turn to *estimating* the two models based on more than one observation, there is not even equivalence with respect to the data constraints, because then the Lagrange multiplier γ is identified and the original first order conditions (2.37) have to be employed which generally do not simplify to (2.10).

Let us briefly summarise what we have found in this subsection: we reviewed the MCCARL's convex-combination-constraints approach which allows to construct a linear model which perfectly aggregates as set of linear programming models based on simulated or observed aggregate activity levels (= aggregate extreme points). This aggregate model can be approximated by a 'frontier' model with a linear objective function subject to a non-linear constraint convex in activity levels. We exemplary showed the equivalence *with respect to calibration* between this frontier model and a non-linear cost function approach for the same functional form under the assumption of identical sets of aggregate resource constraints. However, the frontier model cannot rationalise the non-linear objective function models often used to calibrate programming models with PMP, because it implies a different model structure. Nevertheless, we provided an approach model that constitutes a theoretically equivalent but estimable alternative to convex combination constraints. One cautionary remark with respect to the models ability to compensate for aggregation errors in the context of heterogeneous land: the theoretical derivation of the convex combination constraint relied on the assumption that all activity definitions relevant at farm level are incorporated in the aggregate model. This means, that an aggregate model with convex combination constraints – or with an approximating frontier – can substitute for missing land quality constraints, but not for missing activity definitions. Consequently, a satisfactory application requires data on input-output combinations on different land qualities and the corresponding aggregate land allocations.

2.4.3 Unknown Resource Constraints

Beyond aggregation errors, a second important problem in empirically specifying multi-output programming models is the general lack of information on the availability of fixed resources and the structure of the corresponding constraints. Capital stocks, for example, are often not available or have to be calculated with very extensive and error prone procedures based on investment data (e.g. the perpetual inventory method as in WITZKE 1996), especially if a sufficient degree of differentiation with respect to categories of capital stocks (machinery, buildings or subcategories of those) is desired. If those are obtained, it remains a rather heroic assumption that these quantities can be allocated to the different production activities based upon constant input coefficients. In addition, the typical specification of input coefficients based on engineering information may lead to significantly underused capital stocks rendering an unrealistic non-binding status of the constraints for the data period. In this subsection, we just want to briefly point out that economically relevant but 'empirically missing' resource constraints can be represented by a non-linear transformation function resulting in a programming specification and implied calibration/estimation procedure analogous to the above introduced 'frontier' model (2.32).

An approach closely related to this idea has been employed in the 'European Community Agricultural Model' (ECAM) model. A short description of the supply module is given in MICHALEK (1992), the extensive description with theoretical background can be found in KEYZER (1989) and MERBIS (1989). For the allocation of land to various production activities they maximise overall gross margin (sum of fixed gross margins per hectare) subject to a non-linear 'operating capacity constraint' or 'transformation function', which is convex and non-decreasing in land allocations and accounts for all inputs not explicitly represented in the gross margins. Under the assumption of constant returns to scale they econometrically estimate gross-margin-maximising land shares per unit of operating capacity. The ultimate land allocating programming model is then defined based upon available operating capacity stemming from an exogenous 'investment module' that aggregates available resource capacities in some form. Using an ad-hoc procedure, resulting land allocations are then proportionally adjusted to make the land constraint binding.

A reinterpretation of the frontier model (2.32) might suggest an alternative specification approach based on the same conceptual idea. The constraint $g(\mathbf{x}) = 0$ could be seen as the operating capacity constraint which can be estimated based upon the first order necessary conditions (2.34) and (2.35) given that observations on activity levels (land allocations) and

gross margins are available. The exact specification of the constraint is variable depending on the available information. For example, the parameter 'a' in the quadratic specification (2.36) might represent the operating capacity which could also be set exogenously by some investment module applied to the data period. Alternatively, it could be defined as a function of resource capacities thereby implying a simultaneous estimation of the input aggregator function. The following advantages of this approach to the ECAM specification can be mentioned: First, constant returns to scale assumption with respect to the transformation function is not necessary. Second, adding the land constraint to the specification in (2.32) allows to simultaneously incorporate availability of land and operating capacity during estimation. Third, the approach is conceptually easily extended to accommodate general, non-Leontief type technologies for the various crops. It is the last point that we devote our attention to in the next section.

2.5 Constrained Profit Maximisation under General Technologies

Until now, we have restricted our attention to the typical assumption of Leontief technologies for variable input quantities in programming models, i.e. it was assumed that optimal output and variable input quantities - and thereby gross margins - are constant per unit of crop production activity. We now turn to profit maximisation models that allocate fixed factors to production activities characterised by crop-specific general technologies. It implies that the discourse will more heavily, but not exclusively, draw upon the econometrics literature, which not too long ago, started to estimate behavioural functions, i.e. supply and input demand functions, with explicit representation of fixed factor allocation within the framework of duality theory. The objective is to show that these theoretical concepts can be used to specify and *estimate* equivalent programming models, i.e. simulation models with explicit optimisation but the same underlying model structure as for those behavioural functions.

The basic idea of the general model framework is to characterise the producer's decision making by decomposing the overall optimisation problem into two 'stages': In the first stage, producers maximise profit *given* the allocation of fixed inputs to the various crops. In the second stage, the fixed allocable inputs are distributed across crops to maximise overall profit. CHAMBERS and JUST (1989) first introduced this concept in a seminal article to distinguish between 'truly' joint technologies and apparent jointness due to allocable fixed factors in a profit maximising framework. Subsequent variations of this approach have lead to numerous econometric applications in agricultural supply analysis.

The possibility to explicitly represent allocation of fixed factors, especially land, has two main advantages: First, it introduces an observed decision variable of agricultural producers into the estimation procedure and secondly, it allows the direct representation of factor related premiums such as the per hectare payments of the Common Agricultural Policy (CAP). Therefore, it is not surprising that a considerable amount of applications in this area are in the context of European agricultural crop supply analysis (e.g. GYOMARD, BAUDRY and CARPENTIER 1996; OUDE LANSINK and PEERLINGS 1996; MORO and SCHOKAI 1999; ADENÄUER 1999). In contrast to the typical econometric estimation of output supply and land allocation equations derived from the overall profit maximisation problem, we want to derive first order necessary conditions of various programming models which may then serve as estimating equations for the parameters of these models.

Specifically, we look here at three theoretically equivalent model choices where producers are assumed to maximise profit subject to crop specific production technologies and the allocable factor land:

(1) A model with crop specific profit functions

(2) A model with crop specific cost functions

(3) A model with crop specific production functions.

The three models are only distinguished by the 'degree of explicit optimisation' done by the resulting programming model which increases from (1) to (3).

2.5.1 Crop Specific Profit Functions and Explicit Land Allocation

The first programming model considered is closest to the related econometric literature as it implicit represents variable input allocation and output generation by crop specific dual profit functions, whose parameters are ultimately to be estimated. *Explicit* optimisation only allocates land to the various production activities.

The programming model we want to end up with is of the following structure[13]:

[13] Even though there is some repetition implied, we want to introduce the full notation here again to make the section self contained in this respect.

$$\max_{\mathbf{l}} Z = \sum_{i=1}^{N} \pi^i(p_i, \mathbf{q}, l_i)$$

(2.39) subject to

$$\sum_{i=1}^{N} l_i = b$$

where Z is overall profit, p_i the price of output quantity of the i-th crop $\forall$ i = 1,..., N, $\mathbf{q}$ is a K×1 vector of variable input prices, l_i is the land allocated to the i-th crop, and finally, b is the total available acreage. The functions $\pi^i(p_i,\mathbf{q},l_i)$ are crop specific profit functions defined by

$$\pi^i(p_i, \mathbf{q}, l_i) = \max_{y_i, \mathbf{x}_i} \left[p_i y_i - \sum_{k=1}^{K} q_k x_{ik} : y_i = f^i(\mathbf{x}_i, l_i) \right] \quad \forall i = 1,..., N \tag{2.40}$$

where $y_i = f^i(\mathbf{x}_i, l_i)$ is the production function of the i-th production activity.[14]

In a certain way, equations (2.39) and (2.40) define the optimisation as a two-stage process. In the first stage, the supply quantity of output y_i and variable input demands x_{ij} are determined for a given land allocation l_i based on (2.40). The second stage optimally allocates total land to the different production activities based on (2.39). The latter is our desired programming model and can be written in Lagrangian form as

$$L = \sum_{i=1}^{N} \pi^i(p_i, \mathbf{q}, l_i) + \lambda \left(b - \sum_{i=1}^{N} l_i \right) \tag{2.41}$$

with first order necessary conditions given by[15]

$$\frac{\partial L}{\partial l_i} = \frac{\partial \pi^i(p_i, \mathbf{q}, l_i)}{\partial l_i} - \lambda = 0 \quad \forall i = 1,..., N \tag{2.42}$$

and

$$\frac{\partial L}{\partial \lambda} = b - \sum_{i=1}^{N} l_i = 0 \tag{2.43}$$

where λ is the Lagrange multiplier associated with the total acreage constraint, i.e. the shadow price of land. Note that the two stages are not recursive but simultaneous, because the crop

[14] For a general introduction to dual approaches of supply specification see CHAMBERS 1988.

[15] In order to focus on the principle approach, the representation of all programming models considered in this section assumes strictly positive land allocations to all crops and a binding land constraint. As indicated above, more general cases can be accommodated by adding complementary slackness conditions to the first order conditions. This is also illustrated in the context of a Monte Carlo exercise in section 3.5.

specific profit functions, and consequently the per-unit profits (gross margins), depend on the land allocated to the respective production activities.

The set of equations (2.42) implicitly allocates total land to production activities by equating marginal profit of all production activities to the shadow price of land. For certain functional specifications of $\pi^i(p_i,\mathbf{q},l_i)$, (1.41) and (2.43) can be solved for acreage demand equations of the form

$$l_i = l_i(\mathbf{p},\mathbf{q},b) \quad \forall i = 1,...,N \tag{2.44}$$

where optimal land allocation to a specific crop depends on all output (N×1 vector **p**) and variable input prices as well as availability of land. However, the use of an optimisation algorithm to allocate land and to directly employ first order conditions (2.42) and (2.43) to estimate the parameters of $\pi^i(p_i,\mathbf{q},l_i)$ might imply considerable advantages, because

- it allows to be more flexible with respect to the functional form for and
- political measures (e.g. per ha premiums, base area...etc.) and other potential constraints on land allocation we might want to implement (availability of irrigation water...) complicate if not impede the derivation and estimation of land allocation equations.

The N+1 equations (2.42) and (2.43) are in fact only N independent pieces of information for recovering or estimating the crop specific profit functions, because we usually don't know the shadow price of land. We most often do know the crop quantity supplied and sometimes also the variable input quantities allocated to each crop. In this case we can further employ output supply and input demand equations for given land allocations obtained by applying Hotelling's Lemma to $\pi^i(p_i,\mathbf{q},l_i)$ as

$$y_i = \frac{\partial \pi_i(p_i,\mathbf{q},l_i)}{\partial p_i} = y_i(p_i,\mathbf{q},l_i) \quad \forall i = 1,...,N \tag{2.45}$$

$$x_{ik} = -\frac{\partial \pi_i}{\partial q_k} = x_{ik}(p_i,\mathbf{q},l_i) \quad \forall i = 1,...,N \text{ and } k = 1,...,K. \tag{2.46}$$

Equations (2.42), (2.43), (2.45) and (2.46) or a subset of them could serve as 'data constraints' to simultaneously calibrate or estimate the parameters of all crop specific profit functions (2.40).

A problem with the approach described could be that input allocations to crop activities on the left hand side of (2.46) are often not observed. The crop specific input quantities are typically derived from aggregate quantities based on ad-hoc assumptions combined with engineering

information not consistent with the economic model later specified and used to explain or project these quantities. A theoretically more consistent approach in the context of the current model would be to substitute the (N·K) equations (2.46) by

$$(2.47)\quad x_k = -\sum_{i=1}^{N} \frac{\partial \pi_i}{\partial q_k} = \sum_{i=1}^{N} x_{ik}\left(p_i, \mathbf{q}, l_i\right) \;\forall\; k = 1,...,K.$$

as the total input use can be generally observed at the farm level as well as at the aggregate level. Note that this modified approach does not need input allocations as data but still comes up with crop specific profit functions providing input quantities allocated to each crop.

2.5.2 An Equivalent Cost Function Formulation

The theoretical model presented here relies on the same assumptions as the one in the previous subsection. The only difference is that we separate revenue from cost in formulating the crop specific profit function. This ultimately allows to obtain a final model which not only allocates land but also determines output (or output per ha) through explicit optimisation and requires to calibrate or estimate crop specific cost function parameters for specification.

We can write the crop specific profit functions (2.40) also as

$$(2.48)\quad \begin{aligned} \pi^i(p_i, \mathbf{q}, l_i) &= \max_{y_i, \mathbf{x}_i}\left[p_i y_i - \sum_{k=1}^{K} q_k x_{ik} : y_i = f^i(\mathbf{x}_i, l_i) \right] \\ &= \max_{y_i}\left[p_i y_i - c^i\left(y_i, \mathbf{q}, l_i\right) \right] \end{aligned}$$

where

$$(2.49)\quad c^i(y_i, \mathbf{q}, l_i) = \min_{\mathbf{x}_i}\left[\sum_{k=1}^{K} q_k x_{ik} : y_i = f^i(\mathbf{x}_i, l_i) \right] \quad \forall i = 1,...,N\,.$$

This means that the cost function $c^i(y_i,\mathbf{q},l_i)$ is defined as the minimum variable cost for producing crop quantity y_i, given input price vector $\mathbf{q}$, and land allocation l_i. The considered programming model based upon this cost function representation and equivalent to (2.39) is then defined as

$$(2.50)\quad \begin{aligned} &\max_{y_i, l_i} Z = \sum_{i=1}^{N}\left(p_i y_i - c^i(y_i, \mathbf{q}, l_i) \right) \\ &\text{subject to} \\ &\sum_{i=1}^{N} l_i = b \end{aligned}$$

from which follows the Lagrangian

$$L = \sum_{i=1}^{N}\left(p_i y_i - c^i(y_i, \mathbf{q}, l_i)\right) + \lambda\left(b - \sum_{i=1}^{N} l_i\right) \tag{2.51}$$

and the first order necessary conditions

$$\frac{\partial L}{\partial y_i} = p_i - \frac{\partial c^i\left(y_i, \mathbf{q}, l_i\right)}{\partial y_i} = 0 \quad \forall i = 1,\ldots,N \tag{2.52}$$

$$\frac{\partial L}{\partial l_i} = -\frac{\partial c^i(y_i, \mathbf{q}, l_i)}{\partial l_i} - \lambda = 0 \quad \forall i = 1,\ldots,N\,, \tag{2.53}$$

and the land constraint (2.43) in addition. Conditions (2.52) equate output prices with marginal cost for each use. At first sight, equation (2.53) shows a somewhat surprising definition of the shadow price of land, λ, which should be exactly the same as in the profit function model. It says that the derivative of all cost functions with respect to land multiplied by (–1) must be equal to the shadow price of land. This is nothing but an expression for the marginal opportunity cost of land, i.e. the loss in income (negative marginal cost) caused by substituting other uses of land. Changes in marginal revenue are certainly relevant as well, but they are implicitly adjusted since land allocations also enter (2.52).

In order to see the full equivalence of the shadow price of land in this formulation with the land allocation condition (2.42) from the profit function model, consider the supply function $y^i(p_i,\mathbf{q},l_i)$ implicitly defined by condition (2.52). It solves the maximisation problem (2.48) and appropriately substituted back into the objective function we obtain the known crop specific profit function for given land allocation expressed in terms of the underlying cost and output supply functions:

$$\pi^i(p_i, \mathbf{q}, l_i) = p_i y^i(p_i, \mathbf{q}, l_i) - c^i(y_i(p_i, \mathbf{q}, l_i), \mathbf{q}, l_i) \quad \forall i = 1,\ldots,N\,. \tag{2.54}$$

Differentiating this profit function with respect to land applying the chain rule yields

$$\begin{aligned}
\frac{\partial \pi^i(p_i, \mathbf{q}, l_i)}{\partial l_i} &= p_i \frac{\partial y^i(p_i, \mathbf{q}, l_i)}{\partial l_i} - \left(\frac{\partial c^i(y_i, \mathbf{q}, l_i)}{\partial y_i} \cdot \frac{\partial y^i(p_i, \mathbf{q}, l_i)}{\partial l_i} + \frac{\partial c^i(y_i, \mathbf{q}, l_i)}{\partial l_i}\right) \\
&= \left(p_i - \frac{\partial c^i(y_i, \mathbf{q}, l_i)}{\partial y_i}\right) \cdot \frac{\partial y^i(p_i, \mathbf{q}, l_i)}{\partial l_i} - \frac{\partial c^i(y_i, \mathbf{q}, l_i)}{\partial l_i} \\
&= -\frac{\partial c^i(y_i, \mathbf{q}, l_i)}{\partial l_i} \quad \forall i = 1,\ldots,N
\end{aligned} \tag{2.55}$$

because the term in parentheses in the second line is equal to zero according to (2.52). Consequently, the first order conditions related to land allocation of the profit and the cost

function model, (2.42) and (2.53), are fully equivalent and the shadow price of land is the same in both specifications as expected.

Input demand can be derived using Shephard's Lemma as

$$\frac{\partial c^i(y_i, \mathbf{q}, l_i)}{\partial q_k} = x_{ik} \qquad \forall i = 1,...,N; \forall k = 1,...,K \tag{2.56}$$

so that the land constraint, (2.52), (2.53), and (2.56) or subsets thereof can now be used to simultaneously estimate the parameters of the crop specific cost functions $c^i(y_i,\mathbf{q},l_i)$ and to subsequently specify the programming model (2.50). In case no data on input allocations are available we can substitute (N×K) equations (2.56) by

$$x_k = \sum_{i=1}^{N} \frac{\partial c_i}{\partial q_k} = \sum_{i=1}^{N} x_{ik}(y_i, \mathbf{q}, l_i) \quad \forall k = 1,...,K \tag{2.57}$$

Note once more that the theoretical (i.e. technological as well as behavioural) assumptions of the presented cost function approach are exactly the same as for the preceding profit function approach. The difference is that the cost function approach ends up with an optimisation model which explicitly determines output supply in addition to land allocation and parameters to be estimated are cost function instead of profit function parameters.

We now go one step further and present the same overall model in a framework which allows to recover parameters of crop specific production functions and use those explicitly in the optimisation model.

2.5.3 An Equivalent Production Function Formulation

The optimisation model we want to end up with is simply the primal profit maximisation problem

$$\begin{aligned} &\max_{y_i, x_{ik}, l_i} Z = \sum_{i=1}^{N} p_i y_i - \sum_{i=1}^{N}\sum_{k=1}^{K} q_k x_{ik} \\ &\text{subject to} \\ &y_i = f^i(\mathbf{x}_i, l_i) \quad \forall i = 1,...,N \\ &\sum_{i=1}^{N} l_i = b \end{aligned} \tag{2.58}$$

or, with the production functions $f^i(\mathbf{x}_i,l_i)$ substituted into the objective function,

$$\max_{x_{ik},l_i} Z = \sum_{i=1}^{N} p_i f^i(\mathbf{x}_i, l_i) - \sum_{i=1}^{N}\sum_{k=1}^{K} q_k x_{ik}$$

(2.59) subject to

$$\sum_{i=1}^{N} l_i = b$$

which simultaneously optimises output supply, variable input use, and land allocation. The Lagrangian is given by

$$L = \sum_{i=1}^{N} p_i f^i(\mathbf{x}_i, l_i) - \sum_{i=1}^{N}\sum_{k=1}^{K} q_k x_{ik} + \lambda\left(b - \sum_{i=1}^{N} l_i\right) \tag{2.60}$$

Apart from the land constraint itself, the first order conditions of this optimisation problem imply that marginal value products of variable inputs equal the respective input prices

$$\frac{\partial L}{\partial x_{ik}} = p_i \frac{\partial f^i(\mathbf{x}_i, l_i)}{\partial x_{ik}} - q_k = 0 \quad \forall i = 1,...,N; \forall k = 1,...,K, \tag{2.61}$$

and the equality of land's marginal value product across all uses

$$\frac{\partial L}{\partial l_i} = p_i \frac{\partial f(x_i, l_i)}{\partial l_i} - \lambda = 0 \quad \forall i = 1,...,N, \tag{2.62}$$

Equations (2.61) and (2.62), together with the set of equations calculating supply quantity as

$$y_i = f^i(\mathbf{x}_i, l_i) \quad \forall i = 1,...,N, \tag{2.63}$$

can be used as data constraints to estimate the parameters of the production function. If we do not have observed input allocations x_{ik} but total input quantities x_k, we can treat input allocations like parameters to be estimated, but add the condition

$$x_k = \sum_i x_{ik} \quad \forall k = 1,...,K \tag{2.64}$$

to the data constraints.

Note that the data constraints or estimating equations have the same meaning and informational content, no matter what type of function we want to recover: production functions, cost functions, or profit functions. All models imply profit maximising behaviour and require the same data for calibration or estimation. There is no generally preferable choice of programming model specification with respect to the functional representation. The choice should depend on the specific objective of economic analysis and/or the type of prior information one might have on model parameters.

The underlying profit maximisation framework can be further generalised. The inclusion of more than one allocable fixed factor certainly has empirical relevance and is illustrated based upon a Monte Carlo application of the production function model in chapter 3. Another issue is the accommodation of multi-crop technologies joint in inputs which might be an interesting framework in situations with strong rotational interactions between crops.

2.5.4 A Note on Constrained Profit Maximisation under Multi-Output Technology

Just a brief note shall indicate that the principle approach to specify a programming model under multi-output technologies with input jointness and fixed factors is very similar to the one with crop specific technologies. Joint in inputs implies that they are not allocable to certain outputs but instead affect the production of all outputs. For example, a hectare of land grown with wheat often influences other production processes through effects such as pest interactions, soil productivity etc. If these positive or negative rotational effects are significant, the employment of a multi-output technology representation is more appropriate than crop specific technologies from a theoretical point of view. Nevertheless, although land cannot be exclusively 'allocated' to one output in this case, there is still a difference between growing wheat or sugar for the production of wheat or sugar, i.e. the input land still needs to be distinguished by the observable decision what is grown on it.

We restrict ourselves to a multi-output profit function model with constrained land availability. Keeping notation from the previous section we want to specify the programming model

$$\begin{aligned} &\max_{l_i} Z = \pi(\mathbf{p},\mathbf{q},l_1,...,l_N) \\ &\text{subject to} \\ &\sum_{i=1}^{N} l_i = b \end{aligned} \tag{2.65}$$

which (does not allocate land to crops but) determines profit maximising acreage grown with a certain crop i subject to the land constraint. The multi-output profit function $\pi(\mathbf{p},\mathbf{q},l_1,...,l_N)$ depends on all N output prices $\mathbf{p}$ and 'land uses' $l_1,..., l_N$ and is defined by

$$\pi(\mathbf{p},\mathbf{q},l_1,...,l_N) = \max_{\mathbf{y},\mathbf{x}} \left[\mathbf{p}'\mathbf{y} - \mathbf{q}'\mathbf{x} : T(\mathbf{y},\mathbf{x},l_1,...,l_N) = 0\right] \tag{2.66}$$

where $T(\mathbf{y},\mathbf{x},l_1,...,l_N) = 0$ is the transformation function between outputs, variable inputs and the N different land uses. Contrary to the model with crop specific technologies, the variable inputs quantities are not allocated to outputs and consequently can be represented by a K×1 vector $\mathbf{x}$.

The first order conditions of (2.65) derived from the implied Lagrangian are given by the land constraint and

$$(2.67) \qquad \frac{\partial L}{\partial l_i} = \frac{\partial \pi(\mathbf{p}, \mathbf{q}, l_1, ..., l_N)}{\partial l_i} - \lambda = 0 \qquad \forall i = 1, ..., N .$$

which can be used in combination with output supply and input demand equations derived by Hotelling's Lemma again as equations to estimate the parameters of the – here only one - profit function. Compared to the profit function model with crop specific technologies the estimating equations contain all land uses simultaneously. The cross effects between production processes not only come through the land constraint, but directly enter the partial derivatives of the profit function with respect to l_i. In this respect the current specification is similar to the typical PMP model with the quadratic cost function (2.8).

Just as with the crop specific technologies, we can rewrite model (2.65) also in terms of a cost function or production function model. Since this does not promise any generally new insights we rather take a final look at a variation on the type of assumed objective function in the next section.

2.6 Constrained Expected Utility Maximisation with Price Uncertainty

Risk aversion of agricultural producers is an important reason for output diversification in crop production under stochastic price or yield regimes. Further steps of EU policy reform with decreasing price support and corresponding border protection will make this aspect an even more important issue for European farmers as well as for economic modellers. For this reason, we present a programming model which maximises a linear mean variance utility function subject to a land constraint. It is an extension of earlier work incorporating risk behaviour in agricultural supply analysis by COLLENDER and ZILBERMAN (1985), BABCOCK et al. (1987), CASWELL et al. 1990, COYLE 1992, OUDE LANSINK (1999a), and BOYLE and MCQUINN (2001).[16] It actually combines the approach by OUDE LANSINK with the explicit allocation of the fixed resource land to production activities as presented in the profit maximisation framework above. The model's parameters – including the coefficient of absolute risk aversion - can again be estimated based on the first order conditions which we intend to derive.

[16] See HECKELEI and BRITZ (1998a) for an exploration within the PMP framework.

Let random income ($\tilde{M}$) and expected profit (M) be defined by[17]

$$\tilde{M} = \tilde{\mathbf{p}}'\mathbf{y} - \sum_{i=1}^{N} c^i\left(y_i, \mathbf{q}, l_i\right) \tag{2.68}$$

$$M = \mathbf{p}'\mathbf{y} - \sum_{i=1}^{N} c^i\left(y_i, \mathbf{q}, l_i\right) \tag{2.69}$$

where $\tilde{\mathbf{p}}$ and $\mathbf{p}$ are random and mean output price vectors respectively. All other notation has already been introduced above and the crop specific cost functions $c^i(y_i,\mathbf{q},l_i)$ are still defined according to (2.49). Given that output prices are the only source of uncertainty, the variance of income is given by

$$V = \mathbf{y}'\mathbf{V}_p\mathbf{y} \tag{2.70}$$

with $\mathbf{V}_p$ as the N×N covariance matrix of the price vector $\mathbf{p}$. Expected utility in the linear mean-variance framework is determined by

$$U = M - \frac{1}{2}\alpha V \tag{2.71}$$

where α is the coefficient of absolute risk aversion. If $\alpha > 0$, $\alpha = 0$ and $\alpha < 0$, risk aversion, risk neutrality, and risk affinity are implied, respectively. Substituting (2.69) and (2.70) into the utility function (2.71) and maximising this expression subject to a land constraint results in the programming model specification

$$\begin{aligned} &\max_{y_i, l_i} Z = \mathbf{p}'\mathbf{y} - \sum_{i=1}^{N} c^i\left(y_i, \mathbf{q}, l_i\right) - \frac{1}{2}\alpha\mathbf{y}'\mathbf{V}_p\mathbf{y} \\ &\text{subject to} \\ &\sum_{i=1}^{N} l_i = b \quad [\lambda] \end{aligned} \tag{2.72}$$

Here, a short analogy to the typical PMP model (2.8) with a cost function quadratic in activity levels is worth mentioning: If crop specific cost would not be derived from a general technology but instead be proportional to 'activity level' $\mathbf{y}$, we could express mean income as $\mathbf{p}'\mathbf{y} - \mathbf{d}'\mathbf{y}$. In this case, the matrix $\mathbf{Q}$ in (2.8) could be interpreted as the covariance matrix $\mathbf{V}_p$ scaled by the risk aversion coefficient α. Consequently, the linear mean-variance framework under price uncertainty provides a full rationalisation of quadratic objective function terms that are added to

[17] The introduction of the linear mean-variance utility framework draws heavily upon OUDE LANSINK (1999a). The

the income linear in activity levels. However, a perfect calibration of a multi-output model under Leontief technology is not possible since the covariance matrix of prices is given by the data and the risk aversion parameter α is the only one which can be adjusted.

From (2.72) follows the Lagrangian

$$L = \mathbf{p}'\mathbf{y} - \sum_{i=1}^{N} c^i(y_i, \mathbf{q}, l_i) - \frac{1}{2}\alpha \mathbf{y}'\mathbf{V}_p\mathbf{y} + \lambda\left(b - \sum_{i=1}^{N} l_i\right)$$

and the first order necessary conditions

$$\frac{\partial L}{\partial \mathbf{y}} = \mathbf{p} - \mathbf{MC} - \alpha \mathbf{V}_p\mathbf{y} = 0 \tag{2.73}$$

$$\frac{\partial L}{\partial l_i} = -\frac{\partial c^i(y_i, \mathbf{q}, l_i)}{\partial l_i} - \lambda = 0 \quad \forall i = 1,..., N \tag{2.74}$$

$$\frac{\partial L}{\partial \lambda} = b - \sum_{i=1}^{N} l_i = 0 \quad \forall i = 1,..., N \tag{2.75}$$

where the i^{th} element of the N×1 vector **MC** is defined as $\partial c^i(y_i, \mathbf{q}, l_i)/\partial y_i$. Note, that these first order conditions simplify to the one from the profit maximisation model (2.50) if either risk neutrality ($\alpha = 0$) is assumed or the price variance is zero. Risk aversion ($\alpha > 0$) implies that output prices exceed marginal cost. However, this does not generally imply reduced optimal output quantities compared with profit maximisation. Instead, supply of crops with small price variance will be increased at the expense of high risk crops.

As with the profit maximisation models, a set of estimating equations in addition to the first order conditions can be obtained in form of input demand equations by differentiating the cost functions with respect to input prices. Alternatively, total input quantities as sums of these crop specific input demand functions can be used as data constraints if input allocations are not available. Since these two sets of equations are exactly equivalent to (2.56) and (2.57) they need not be repeated.

Finally, it shall be mentioned that the model can be changed to represent uncertainty in yields or prices *and* yields, which might often be even more relevant. However, for the purpose of this study, a sufficient overview on potentially relevant programming models for agricultural supply analysis has been given. It is time to complete the suggested approach for the calibration or

representation with *crop specific* cost functions has been employed by BOYLE and MCQUINN (2001) as well.

estimation of programming models based on first order conditions with the actual methodological implementation in the subsequent chapter.

2.7 Summary of Chapter 2

This chapter dealt with the theory underlying calibration and estimation of programming models for agricultural (crop) supply analysis, i.e. models which explicitly optimise some economic objective function subject to constraints. The current most prominent approach in this context, 'Positive Mathematical Programming' (PMP), was reviewed. The procedure employs dual values of calibration constraints to specify additional non-linear terms in the objective function, which are interpreted as hidden marginal cost or marginal revenues not represented by the available data. Most applications are based upon just one base year observation on production program and economic indicators (calibration case). Various ad-hoc assumptions to solve the generally implied underdetermined parameter specification problem render models with very different simulation response. It was argued that only the introduction of prior information such as exogenous elasticities and/or parameter *estimation* using multiple observations provide satisfactory solutions. However, the structural discrepancy between the 'first-phase-model' and the resulting model when applying PMP was shown to base estimation on inconsistent data constraints.

Then, a simple but general alternative approach to calibrate or estimate programming models was introduced which uses first order necessary conditions derived from the desired model structure as estimating equations. This approach is not only suitable for typical PMP models but can be applied to general programming model specifications as well. It has the potential to provide an equivalent and at the same time more flexible way for estimating constrained optimisation models compared to traditional econometric approaches based on duality theory. Therefore, the remainder of the chapter derived first order conditions of various supply models stemming from the programming and econometric literature that could be used to estimate parametric specifications of these models:

First, rationalisation of non-linear terms in profit maximisation models with Leontief technology were considered. The inclusion of crop specific land supply functions to represent land heterogeneity was shown to result in a programming model equivalent to PMP models with non-linear costs in activity levels. A more general look at the underlying aggregation problem if the distribution of land qualities is not available lead to a variant of 'convex combination constraints'

where a frontier of extreme points is approximated by a non-linear constraint. The same specification can be interpreted as representing a set of unknown constraints.

Second, profit maximisation models under general technologies with allocation of the fixed factor land were examined. They had been extensively used in the recent past to econometrically estimate supply and land allocation equations under the EU crop policy regime. In the context of crop specific technologies, three equivalent models provide the possibility to alternatively estimate the parameters of profit-, cost- or production functions for each crop while maintaining the same economic structure and requiring identical data information. A brief note illustrated the generalisation of the framework to multi-output technologies.

Third, the profit maximisation hypothesis was abandoned to explicitly incorporate risk behaviour on the side of agricultural producers. The expected utility maximisation model under price risk and with explicit land allocations had been used before as well for the econometric estimation of behavioural functions. The employment as a programming model estimated based upon the relevant first order conditions potentially opens the way for more complex but nevertheless consistently estimated specifications within this framework.

The next chapter is now responsible for putting the theory into practice. Exemplary applications of the approach shall demonstrate its functionality by evaluating estimation exercises with Monte Carlo simulations and by providing successful calibrations to observed base year observations.

3 Methodological Approaches to the Calibration and Estimation of Programming Models

3.1 Introduction to Chapter 3

Chapter 2 argued for a general and theoretically consistent alternative to PMP in calibrating and estimating agricultural programming models based on first order conditions of the desired model specification and without the use of dual values on calibration constraints. Furthermore, the approach promises to be theoretically equivalent, but empirically more flexible than previous dual econometric models with explicit allocation of fixed factors.

It is the major objective of this part to further develop and illustrate *methodological* concepts to calibrate and estimate specifications of agricultural programming models based on these principles. The specific objectives are (1) to show the general feasibility of the approach using concrete specifications of three different types of programming models introduced in the previous chapter, (2) to evaluate the estimation principle using the data-constrained Generalised Maximum Entropy approach (GME-D, GOLAN et al. 1996) in this context with Monte Carlo simulation techniques, and (3) to develop empirically feasible approaches for including prior, i.e. non-sample information to improve finite sample accuracy of the estimates.

The idea to use multiple observations for the specification of programming models was already suggested by PARIS and HOWITT 1998 when they introduced the Maximum Entropy procedure in the PMP context. Empirical applications with more than one observation, however, have been limited to a cross sectional estimation by HECKELEI and BRITZ (2000) and PARIS (2001). To our knowledge, this study is the first experimental analysis of the specification of programming models with GME-D techniques.

The presentation is organised as follows: Section 3.2 gives an introduction to the GME-D approach in the context of the general linear model. It serves to give some intuitive insight to this estimation mechanism but also summarises relevant properties from the literature. Sections 3.3 to 3.5 look at three different model specifications for crop supply behaviour under profit maximisation, first at the case of a typical PMP model specification with Leontief production technologies, one allocable fixed factor and a cost function quadratic in activity levels (QP-model), second at a model with two allocable fixed factors and production technologies of the Constant-Elasticity-of-Substitution- (CES) type, and third at a duality based specification with normalised quadratic (NQ), crop specific profit functions. Each of these sections give a short

model overview. Then, a Monte Carlo experimental set-up is introduced with noise components in the data generation process and allows to evaluate the estimation approach for multiple observations in a repeated sampling context. It is subsequently used to evaluate the general estimation accuracy of the approach with respect to model parameters across different sample sizes. In addition, concepts for the inclusion of prior information on elasticities and shadow prices are introduced and their effects are investigated (for the QP and CES model). Furthermore, the possibility to estimate across the binding and non-binding status of inequality restrictions is considered (NQ profit function model). Section 3.6 concludes with a summary of the main results.

The reader should be warned already here of the somewhat lengthy and cumbersome mathematical notation in this chapter, especially in the context of the Monte Carlo experiments. This is partially due to the complexity of the optimisation models considered and the nature of the GME-D approach employed. However, the main reason is the decision on the side of the author to omit generic representations of equations in order ease potential trials of readers to implement these or similar procedures for their own research.

3.2 Data-Constrained Generalised Maximum Entropy Approach

This chapter presents calibration and estimation approaches of empirical models using the Maximum Entropy Criterion. Specifically, the *data constrained generalised maximum entropy* approach (GME-D), which was introduced by GOLAN et al. in 1996 to a wider range of economists. This approach has been chosen because,

(1) it is equally applicable to ill-posed and well-posed inference problems, and

(2) it provides a straightforward way of incorporating prior information on model parameters.

Point (1) allows to use the same general methodology no matter whether the number of observations exceeds the number of parameters to be estimated (well-posed) or not (ill-posed)[18]. If the number of observations is small, i.e. data information scarce, prior information on model parameters becomes a valuable, if not a necessary ingredient to obtain a sensible model specification. But beyond cases of extreme data scarcity, the use of relevant prior information (non-sample information) might considerably improve the finite sample properties of estimators for a wide range of empirical models.

[18] MITTELHAMMER et al. (2000) include the cases of mutually inconsistent observations and an ill-conditioned variable matrix **X** in the definition of 'ill-posed '. We stick to the more restrictive definition in our context.

Although the two arguments mentioned in support of the GME-D approach can probably be considered relevant for the estimation of many empirical models, they are especially important in the context of this study: policy relevant agricultural supply models typically require a highly differentiated specification with respect to outputs, inputs and/or regional dimension. The cost to compile such a database is very high and often severely limits the data range in the cross sectional or time domain. The extreme - but still frequent - case is the calibration of a model to one point in time based on exogenous elasticities (or other ad-hoc approaches, see chapter 2).

In addition, to achieve acceptance of the model by political decision makers, it is often required to make the model consistent with available prior information. This has nothing to do with 'manufacturing' model results according to political convenience or prior beliefs. Instead, it acknowledges the fact that limitations in theoretical model specification, data availability, and estimation methodology can result in clearly unrealistic empirical models. The GME-D approach allows to address these problems by using prior information in a mathematically exact and *reproducible* way. It provides in fact the opportunity to improve the data contribution to the specification of policy relevant models, because it can replace *ad-hoc* and *synthetic* approaches to applied modelling with a consistent estimation methodology that makes best use of all the available information.[19]

Note: PARIS and HOWITT (1998, 2001) and PARIS (2001) also make use of Maximum Entropy techniques in specifying programming models and we follow their path in this respect. However, no effort is made to develop or use approaches that claim to be purely data based or with minimal prior information in the case of minimal data information (see for example VAN ACKEREN et al. 2001 who define a purely data based GME estimator). In our view, any attempt to specify realistic models of economic behaviour under changing conditions with less than 5 observations and without using prior information is doomed to fail.

The issues mentioned above will be picked up again in the context of the various model specifications to provide more than just a general discussion. Before we move to the more complex models represented by first order conditions of programming problems we want to present the principles of the GME-D approach in the context of the well known single equation linear regression model. A discussion on issues relevant for finite sample properties follows and

[19] The traditional econometric approach to incorporate prior information employs Bayes theorem (see for example HECKELEI and MITTELHAMMER (1996 and 2002) for an introduction in the context of single and multivariate regression and a semiparametric extension and HECKELEI (1995) and HECKELEI and BRITZ (1998b) for an

the exposition of asymptotic properties conclude the GME-D introduction. For a more general and extensive exposition we refer to the original source GOLAN et al. (1996) or to MITTELHAMMER et al. (2000, chapter E3) to obtain a detailed introduction within the context of a recent econometrics textbook.

3.2.1 Estimation of the General Linear Model with GME-D

Consider the standard linear regression model

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e} \tag{3.1}$$

where $\mathbf{y}$ is a N×1 vector of observations on the dependent variable, $\mathbf{X}$ a N×K matrix of explanatory variables with full rank, and $\boldsymbol{\beta}$ a unknown K×1 vector of regression coefficients. The unobservable N×1 vector of random independent identically distributed (iid) disturbances $\mathbf{e}$ has mean $E[\mathbf{e}] = \mathbf{0}$ and covariance matrix $E[\mathbf{ee}'] = \sigma^2\mathbf{I}$.

If no prior information on $\boldsymbol{\beta}$ exist and the problem is well-posed (N>K) we can use Ordinary Least Squares (OLS) to obtain estimates of $\boldsymbol{\beta}$ as $\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$. It is well known that under these circumstances, $\mathbf{b}$ is the best linear unbiased estimator of $\boldsymbol{\beta}$ (Gauss Markov Theorem, see for example MITTELHAMMER 1996: 436).

In order to apply GME-D, we need to reparameterise equation (3.1) such that each element of the vectors $\boldsymbol{\beta}$ and $\mathbf{e}$ is expressed as an expectation of a discrete probability distribution. Let $\mathbf{Z}$ be a block-diagonal K×KS matrix of *support points*, i.e. mutually exclusive and exhaustive events of K discrete random variables, and $\mathbf{p}$ be the corresponding KS×1 vector of probabilities. The vector $\boldsymbol{\beta}$ can then be expressed as

$$\boldsymbol{\beta} = \mathbf{Z}\mathbf{p} = \begin{bmatrix} \mathbf{z}_1' & 0 & \cdots & 0 \\ 0 & \mathbf{z}_2' & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \mathbf{z}_K' \end{bmatrix} \begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \vdots \\ \mathbf{p}_K \end{bmatrix} \tag{3.2}$$

with $\mathbf{z}_k' = \begin{bmatrix} z_{k1} & z_{k2} & \cdots & z_{kS} \end{bmatrix}$ such that $z_{k1} < z_{k2} < \ldots < z_{kS}$. Accordingly, the N×NS block-diagonal matrix of support points $\mathbf{V}$ and the corresponding NS×1 vector of probabilities $\mathbf{w}$ allow to write

application to a demand and supply specification, respectively). However, most implementations require positive degrees of freedom of the estimation problem.

(3.3) $$\mathbf{e} = \mathbf{V}\mathbf{w} = \begin{bmatrix} \mathbf{v}_1' & 0 & \cdots & 0 \\ 0 & \mathbf{v}_2' & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \mathbf{v}_N' \end{bmatrix} \begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \\ \vdots \\ \mathbf{w}_N \end{bmatrix}$$

with $\mathbf{v}_n' = \begin{bmatrix} v_{n1} & v_{n2} & \cdots & v_{nS} \end{bmatrix}$ such that $v_{n1} < v_{n2} < \ldots < v_{nS}$.

We can now write the reparameterised version of (3.1) as

(3.4) $$\mathbf{y} = \mathbf{X}\mathbf{Z}\mathbf{p} + \mathbf{V}\mathbf{w}$$

which constitutes the 'data constraints' of the GME-D approach. Realising that the elements of each $\mathbf{p}_k$, k = 1,...,K, and $\mathbf{w}_n$, n = 1,...,N sum up to 1 according to their definition as probabilities, equations (3.2) and (3.3) define the admissible values for the elements of $\boldsymbol{\beta}$ and $\mathbf{e}$ as convex combinations of the corresponding support points $\mathbf{z}_k$, k = 1,...,K, and $\mathbf{v}_n$, n = 1,...,N. This implies that the range of support points $[z_{k1}, z_{kS}]$ and $[v_{n1}, v_{nS}]$ limits the associated values $\boldsymbol{\beta}_k$ and $\mathbf{v}_n$ to the same interval.

The GME-D approach now chooses among the infinite number of vectors $\mathbf{p}$ and $\mathbf{w}$ satisfying (3.4) for predetermined $\mathbf{Z}$ and $\mathbf{V}$ as the ones which maximises the Entropy criterion

(3.5) $$H[\mathbf{p}, \mathbf{w}] = -\mathbf{p}' \ln \mathbf{p} - \mathbf{w}' \ln \mathbf{w}$$

where the logarithm of zero probabilities is defined as ln(0) = 0. This objective function reaches its unconstrained maximum where all elements of $\mathbf{p}$ and $\mathbf{w}$ have the value 1/M, i.e. when the probabilities are uniform. Since the uniform distribution treats each outcome equally likely one can view this distribution as the maximally uninformed distribution with respect to anticipating outcomes of a random variable. 'Thus, the *maximum value of entropy* is uniquely associated with the *maximally uninformative weight- probability distribution*" (MITTELHAMMER et al. 2000, E3: 8).

We can now state the complete estimation problem as

$$\max_{\mathbf{p},\mathbf{w}} H(\mathbf{p}, \mathbf{w}) = -\mathbf{p}' \ln \mathbf{p} - \mathbf{w}' \ln \mathbf{w}$$

subject to

(3.6) $$\mathbf{y} = \mathbf{X}\mathbf{Z}\mathbf{p} + \mathbf{V}\mathbf{w}$$

$$\mathbf{u}'\mathbf{p}_k = 1 \quad \forall k$$

$$\mathbf{u}'\mathbf{w}_n = 1 \quad \forall n$$

where the last two constraints ensure that the probabilities appropriately add up to one and **u** is a S×1 'summation vector', i.e. a conformable vector of ones.[20] The values of $\boldsymbol{\beta}$ and **e** can be recovered by the definitions (3.2) and (3.3).

3.2.2 Issues Relevant for Finite Sample Properties of the GME-D Approach

There is ample Monte-Carlo evidence in the literature, that the GME-D estimator can outperform appropriate classical estimators in a variety of data sampling situations with respect to estimation accuracy in finite samples. Naturally, this evidence is stronger under conditions difficult to handle for traditional approaches such as high multicollinearity of the columns of **X** or in the case of nearly unidentified simultaneous equation systems[21]. The support points **Z** and **V**, however, play a critical role for the efficiency with which the parameter vector $\boldsymbol{\beta}$ can be estimated. In contrast to most published Monte Carlo simulations which employ a more or less arbitrary rule to define **Z** and **V**, we want to explicitly use **Z** to define prior information on the parameters or functions thereof and investigate how the broadened information base affects estimator properties. For a better understanding of the mechanisms, the following issues are relevant:

It is evident from (3.6) that only the N data constraints (3.4) potentially keep the elements of **p** and **w** from taking on uniform values. This does not imply that information on the actual model parameters is only provided by the data. In fact, there are specific values of $\boldsymbol{\beta}$ and **e** associated with the most uninformative – uniform – distribution of probabilities **p** and **w**, which can be viewed as *prior expectations*. Those values are recovered as point estimates from (3.6) if the data constraints do not provide any counter-evidence, i.e. are satisfied at these values. In the typical case of equidistant support points, these prior expectations are the centre of the support ranges.

The implied prior expectation is not the only relevant issue with respect to the specification of support points. The *range of the supports* determines how strongly the entropy criterion 'pulls' towards the prior expectation. Consequently, by adjusting the range one can express relative uncertainty (wide range) and relative certainty (narrow range) with respect to the prior information on model parameters.

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[20] Throughout the remainder of this study, the 'summation vector **u**' adapts automatically to the dimensions of the post-multiplied vector or matrix without special mention in the text.

[21] GOLAN et al. (1996) provide results of Monte Carlo experiments for many different versions of estimators. More systematic simulations with respect to specific estimators can be found, for example, in VAN ACKEREN et al. (2001), MARSH et al. (2001).

This important role of the support points in the GME-D approach constitutes danger and potential at the same time. Danger, because poor choices of the prior expectations and support range may introduce a strong estimation bias if the data information is scarce. Potential, because it allows to flexibly incorporate valid prior information on model parameters which increases the information base and generally decreases the variance of the estimates. Emphasis on the former lead to attempts in defining 'data based' support points (VAN ACKEREN et al. 2001). Emphasis on the latter lead to a widespread use of GME-D in a variety of applied calibration and estimation problems in agricultural economics which could not be handled in such an elegant and reproducible manner before (e.g. LENCE and MILLER 1998a and b; LÉON et al. 1999; OUDE LANSINK, 1999b; ZHANG and FAN 2001). The Monte Carlo simulations in the context of estimating programming models presented in the subsequent sections put a strong emphasis on the trade-off between bias and variance of the estimates.

It shall be mentioned here, that there exist more general formulations of entropy-based objective functions than the one employed in (3.6). A particularly relevant one for the incorporation of prior information is the *Weighted Generalized Cross Entropy* (GCE(γ)) criterion (GOLAN et al. 1996: 111) which minimises

$$\text{(3.7)} \qquad I[\mathbf{p},\mathbf{w}] = (1-\gamma)\mathbf{p}'\ln(\mathbf{p}/\mathbf{q}) + \gamma\mathbf{w}'\ln(\mathbf{w}/\mathbf{r})$$

where **q** and **r** are conformable prior probabilities on the support points in **Z** and **V** and $\gamma \in (0,1)$ is a weight which allows to adjust the relative importance of error and parameter entropy. The unconstrained objective function (3.7) obtains its minimum when **p** = **q** and **w** = **r**. This means for the data constrained case that the criterion draws to the prior distribution as much as the data constraints allow. The minimisation of I[**p**,**w**] is equivalent to the maximisation of H[**p**,**w**] if all elements in **q** and **r** are equal to 1/M and $\gamma = 0.5$.

The impact of adjusting the parameter γ is not as clear as it might seem at first sight. Increasing γ intuitively suggests that the deviation of **w** from **r** becomes relatively less desirable so that the influence of prior information on parameters is reduced in turn. GOLAN et al. (1996: 112-114) show that this conjecture is not true in general. Further Monte Carlo evidence indicate a positive impact of 'optimal' γ on finite sample properties, but no general – data independent – conclusions can be drawn (see VAN ACKEREN et al. 2001).

In the current study, we restrict our attention for the most part to the simple entropy objective function (3.5). We completely ignore the possibility of defining prior information via the prior probability weights **q** and **u** leaving the investigation of comparative advantages to future

research. We do, however, employ estimators without parameter supports in well posed situations, which is equivalent to setting $\gamma = 0$.

Apart from formulating prior information, a further choice with respect to the support points has to be made which is potentially relevant for the finite sample properties: The determination of S, i.e. the number of support points. GOLAN et al. (1996:139) show that the variance of the estimator decreases as S increases approaching a limit that depends on the data and the support range. The simulations in the subsequent sections do not further investigate the relevance of this issue. Simulation tests by the author as well as GOLAN et al. revealed that the variance reduction is generally irrelevant beyond 4 support points for all practical purposes. More importantly, the range of the support points determined by the prior information is a strongly dominant factor with respect to the estimator's variability even at S = 2. Given the relative small impact and the need to reduce computational demands, we let S = 2 in all GME-D applications of this chapter.

3.2.3 Asymptotic Properties of the GME-D Approach

MITTELHAMMER and CARDELL (2000) derive consistency and asymptotic normality for the GME-D estimator under a set of regularity conditions. The following assumptions must hold in addition to the GLM assumptions introduced in the context of (3.1):

(1) The true domain of e_i is contained in some large enough closed interval

(2) The probability density function (pdf) of e_i is symmetric around zero

(3) The true parameters lie within the support range, i.e $\beta_k \in (z_{k1}, z_{kS})$

The consistency property is especially comforting if one wants to use prior information on the parameters. One can be sure that the generally introduced bias goes away with increasing data information. Asymptotic normality implies the general advantage of having standard inference procedures available which are at least asymptotically valid (see, for example, MITTELHAMMER et al. 2000, E3: 42-45).

Having introduced the asymptotic properties of the GME-D approach in the context of the multiple linear regression model, a cautionary remark is in order with respect to the applications below. Consistency and asymptotic normality of the estimates cannot automatically be assumed because several deviations occur (not necessarily all in each application):

- Equations are non-linear in parameters[22]
- Equations contain more than one error term
- Existence of 'free' parameters, i.e. without associated supports
- Existence of supports on functions of parameters

The generalisation of asymptotic GME-D properties to these cases is beyond the scope of this research project, because GME-D is merely employed as a useful tool for the specification of agricultural programming models in small samples. However, variations in the numbers of observations and heuristic arguments are provided to give at least strong hints with respect to consistent behaviour in the questionable situations.

3.3 Land Allocation with Quadratic Cost Function

3.3.1 Model Overview and Estimation Approach

This subsection deals with estimating the parameters of the optimisation model employing a quadratic cost function often used in the PMP context and already described chapter 2 (model 2.8). For the sake of simplicity we just consider the resource land as fixed at b rendering a quadratic programming model (QP-model) with one constraint and a corresponding scalar shadow price λ. In addition, we replace the vector of prices **p** by a vector of gross margins **gm**[23] to obtain

$$\begin{aligned} &\max_{\mathbf{l}} Z = \mathbf{gm}'\mathbf{l} - \mathbf{d}'\mathbf{l} - 0.5\mathbf{l}'\mathbf{Q}\mathbf{l} \\ &\text{subject to} \qquad\qquad , \\ &\mathbf{u}'\mathbf{l} \le b \;\; [\lambda], \;\; \mathbf{l} \ge \mathbf{0} \end{aligned} \tag{3.8}$$

with **l** being the vector of land allocations, **d** and **Q** representing (N×1) parameter vector and an (N×N) parameter matrix, respectively.

If we assume that the optimal land allocations satisfy the land constraint in equality we obtain the first order conditions of (3.8) as

[22] Only for a special case of a non-linear model a consistency proof is given in the literature (GOLAN, JUDGE and PERLOFF 1996)

[23] The quadratic cost function represents 'some' unknown non-linear cost, which are independent of the variable inputs per activity unit. This lack of rationalisation in the model is analogous to many PMP applications and has been discussed in chapter 2. The illustration based on the model shall by no means indicate the preferability of this model.

(3.9) $$\mathbf{gm} - \lambda\mathbf{u} - \mathbf{d} - \mathbf{Ql} = 0$$

(3.10) $$\mathbf{u}'\mathbf{l} = b\,.$$

Denoting all observed variables by the superscript 'o', realising that (3.9) and (3.10) have to be satisfied for every observation t = 1,...,T, and further assuming that observed land allocations, $\mathbf{l}_t^o$, are obtained from optimal values by adding an (N×1) vector of stochastic errors $\mathbf{e}_t$ with mean zero and standard deviation σ_i, we can write the first order conditions in terms of data constraints as

(3.11) $$\mathbf{gm}_t^o - \lambda_t\mathbf{u} - \mathbf{d} - \mathbf{Q}\left(\mathbf{l}_t^o - \mathbf{e}_t\right) = 0 \quad \forall t$$

(3.12) $$\mathbf{u}'\left(\mathbf{l}_t^o - \mathbf{e}_t\right) = b_t^o \quad \forall t\,.$$

As an estimation technique, we employ the Generalised Maximum Entropy GME approach introduced above, which allows a flexible incorporation of out of sample information further below.[24] For now, however, we restrict support point definitions to the errors. We reparameterise the error vectors as expected values of a discrete probability distribution. The (N×(2N)) Matrix **V** with S = 2 support points for each error term bounds the support to +/-5 standard deviations.[25] For the simulation experiments below we have N = 3 crops so that the error terms can be represented as the multiplication of **V** with a ((3·2)×1)-vector of probabilities $\mathbf{w}_t$ to obtain

(3.13) $$\mathbf{e}_t = \mathbf{V}\mathbf{w}_t = \begin{bmatrix} -5\sigma_1 & 5\sigma_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -5\sigma_2 & 5\sigma_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5\sigma_3 & 5\sigma_3 \end{bmatrix} \begin{bmatrix} w_{11t} \\ w_{12t} \\ w_{21t} \\ w_{22t} \\ w_{31t} \\ w_{32t} \end{bmatrix}.$$

The complete GME formulation is then[26]

(3.14) $$\max_{\mathbf{w}_t,\mathbf{Q},\mathbf{L},\lambda_t} H\left(\mathbf{w}_t\right) = -\sum_{t=1}^{T} \mathbf{w}_t' \ln \mathbf{w}_t$$

[24] It shall be mentioned, however, that in this context of 'well-posed' estimation problems with more observations than parameters to be estimated, classical techniques such as least squares could have been applied as well.

[25] With respect to the support range GOLAN et al. (1996) suggest the '3-Sigma' rule. PRECKEL (2001) advocates a rather large range to approximate the behaviour of the least squares estimator.

[26] For the current case of just one resource constraint the vector **d** is not identified. Therefore, its elements are set to zero.

subject to

(3.15) $$\mathbf{gm}_t^o - \lambda_t \mathbf{u} - \mathbf{Q}\left(\mathbf{l}_t^o - \mathbf{V}\mathbf{w}_t\right) = 0 \quad \forall t$$

(3.16) $$\mathbf{u}'\left(\mathbf{l}_t^o - \mathbf{V}\mathbf{w}_t\right) = b_t^o \quad \forall t$$

(3.17) $$\mathbf{Q} = \mathbf{L}\mathbf{L}' \text{ mit } L_{ij} = 0 \ \forall \ j > i$$

(3.18) $$\sum_{s=1}^{S} w_{its} = 1 \quad \forall i, t$$

where $H(\mathbf{w}_t)$ denotes Entropy, equation (3.17) guarantees the positive (semi-) definiteness of $\mathbf{Q}$ based on a Cholesky factorisation, and (3.18) ensures that the probabilities add up to one. Note, that we do not need any reparameterisation of model parameters, because we only consider 'well-posed' problems with positive degrees of freedom in our simulations.

3.3.2 Monte Carlo Simulations without Prior Information

The following Monte Carlo simulation experiment is used to test the estimators precision: Based on the output and input differentiation in HOWITT (1995b) a data set with T observations is generated for T different random vectors $\mathbf{gm}_t$ and $\mathbf{b}_t$ for given parameters $\mathbf{Q}$.[27] Normally distributed errors are added to the optimal land allocations l_{1t}^* and l_{2t}^* of the first two crops with a standard deviation of 2% of the average land allocation, so that the 'observed' allocations are calculated as $l_{1t}^o = l_{1t}^* + e_{1t}$ and $l_{2t}^o = l_{2t}^* + e_{2t}$. To ensure that the land restriction is binding at the observed production activity levels we let $l_{3t}^o = b_t - l_{1t}^* - l_{2t}^*$.

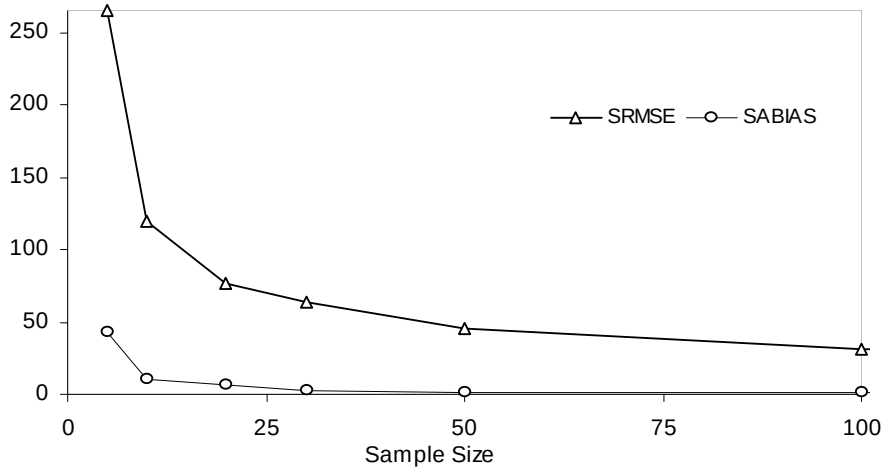
For every generated data set, the model parameters are estimated with the GME approach and the whole procedure is repeated 1000 times for each sample size (see QPSIM.GMS in appendix).[28] The quality of the estimation is evaluated using the measures absolute bias ('ABIAS' = absolute value of the difference between average estimate and true value of the parameter) and

[27] See section A3.1 in the appendix of this chapter for the basic data set from HOWITT which reflects the values of non-random model variables and the means of random variables of all simulation exercises presented in this chapter. The parameter values of all 'true' models and additional parametric restrictions included during estimation are reported in sections A3.2 to A3.4 of the appendix as well.

[28] All estimation and most calibration results in this chapter are obtained using the General Algebraic Modelling System (GAMS, BROOKE et.al. 1988). GAMS programs for Monte Carlo simulations and calibration approaches are provided for in section A3.6 of the appendix to this chapter. This shall document the work in the most precise and reproducible way possible.

root mean square error ('RMSE'). For a representative look at the results the measures are summed over all estimated parameters (here all elements of **Q**).

Figure 3.1 QP-Model – SRMSE and SABIAS without Prior Information



So

Source: Own calculations.

Figure 3.1 presents the results for different sample sizes. The sum of all RMSE (SRMSE) decreases with increasing sample size indicating consistency of the estimator. The sum of absolute biases quickly decreases to irrelevant values already at a sample size of 20 (SABIAS). Recalling that the MSE is the sum of the squared bias and the empirical variance, Figure 3.1 shows that the bias reflects a small fraction of the RMSE only and the much more important part of the MSE is given by the standard errors of the estimates. For small sample sizes - which are often encountered in empirical work for differentiated analyses - this could obviously result in very poor estimates. In this case the use of out of sample information is a potential remedy. Ideally, the employment of prior information would reduce the estimators variance at small sample sizes without introducing a strong additional bias. To get a better feel for the required precision of the prior information and the general interplay between prior and data in our modelling context we further extended the simulations:

3.3.3 Monte Carlo Simulations with Prior Information on Supply Elasticities

An empirically relevant possibility for incorporating out of sample information is the use of priors on elasticities. Other studies with comparable objectives frequently provide at least a general idea on their range. A reparameterisation of these elasticites analogous to the one for the error terms allows the technical representation. For the current model, we can employ the following analytical expression for the (N×1) vector of land allocation elasticities with respect to

own gross margins $\boldsymbol{\varepsilon}$ based on the marginal effects on activity levels derived in equation (2.15) and here adjusted for the case of a single land constraint :

$$\begin{aligned}\boldsymbol{\varepsilon} &= \operatorname{diag}\left(\frac{\partial \mathbf{l}}{\partial \mathbf{gm}} \odot \left[\frac{\overline{\mathbf{gm}}}{\bar{\mathbf{l}}^{o}}\right]'\right) \\ &= \operatorname{diag}\left(\left(\mathbf{Q}^{-1} - \mathbf{Q}^{-1}\mathbf{u}\left(\mathbf{u}'\mathbf{Q}^{-1}\mathbf{u}\right)^{-1}\mathbf{u}'\mathbf{Q}^{-1}\right) \odot \left[\frac{\overline{\mathbf{gm}}}{\bar{\mathbf{l}}^{o}}\right]'\right)\end{aligned} \tag{3.19}$$

where $\left[\partial \mathbf{l}/\partial \mathbf{gm}\right]$ represents the (N×N) Jacobian matrix of the land demand functions and the i,j-th element of the (N×N) matrix $\left[\overline{\mathbf{gm}}/\bar{\mathbf{l}}^{o}\right]$ is defined as the sample mean of gross margin i, $\overline{gm_i}$, divided by the sample mean of observed land allocation to crop j, $\bar{l}_j^o$. Combined with the elasticity reparameterisation we have to add the constraint

$$\mathbf{V}^{\varepsilon}\mathbf{w}^{\varepsilon} = \operatorname{diag}\left(\left(\mathbf{Q}^{-1} - \mathbf{Q}^{-1}\mathbf{u}\left(\mathbf{u}'\mathbf{Q}^{-1}\mathbf{u}\right)^{-1}\mathbf{u}'\mathbf{Q}^{-1}\right) \odot \left[\frac{\overline{\mathbf{gm}}}{\bar{\mathbf{l}}^{o}}\right]'\right) \tag{3.20}$$

with

$$\mathbf{V}^{\varepsilon} = \begin{bmatrix} v_{11}^{\varepsilon} & v_{12}^{\varepsilon} & 0 & 0 & 0 & 0 \\ 0 & 0 & v_{21}^{\varepsilon} & v_{22}^{\varepsilon} & 0 & 0 \\ 0 & 0 & 0 & 0 & v_{31}^{\varepsilon} & v_{32}^{\varepsilon} \end{bmatrix} \quad \text{and} \quad \mathbf{w}^{\varepsilon} = \begin{bmatrix} w_{11}^{\varepsilon} \\ w_{12}^{\varepsilon} \\ w_{21}^{\varepsilon} \\ w_{22}^{\varepsilon} \\ w_{31}^{\varepsilon} \\ w_{32}^{\varepsilon} \end{bmatrix} \tag{3.21}$$

to the previous (3.15) - (3.18), where v_{i1}^{ε} and v_{i2}^{ε} are the lower and upper support points of the i-th elasticity, respectively, and w_{i1}^{ε} and w_{i2}^{ε} the corresponding probabilities. The objective function has to be modified to

$$\max_{\mathbf{w}_t, \mathbf{w}^{\varepsilon}, \mathbf{Q}, \mathbf{L}, \lambda_t} H\left(\mathbf{w}_t, \mathbf{w}^{\varepsilon}\right) = -\sum_{t=1}^{T} \mathbf{w}_t' \ln \mathbf{w}_t - \mathbf{w}^{\varepsilon\prime} \ln \mathbf{w}^{\varepsilon}. \tag{3.22}$$

The intuition behind the objective function is as follows: the entropy criterion generally pulls towards the centre of the elasticity supports in trade-off with the error terms of the data constraints. The smaller the range of the elasticity support the higher is the penalty for deviating from the support centre. Consequently, the variation of the support range allows to express the precision of the a-priori information. A necessary condition for consistency, however, is that the

true elasticity remains within the support range. Only then it is possible that the increasing weight of the error probabilities in the objective function draws the parameter estimates to their true values as more observations become available.

The approach is analogous to the typical procedure in the framework of GME, but standard theoretical exposition (e.g. GOLAN et al. 1996) and agricultural economics applications (e.g. LENCE and MILLER 1998a and b; LÉON et al. 1999, OUDE LANSINK 1999b; ZHANG and FAN 2001) so far only employed direct restrictions on the parameter space to make the approach suitable for ill-posed and ill-conditioned problems. The restrictions on *functions* of parameters used here, however, are often more appropriate to incorporate available out of sample information.

After these technical remarks we come to the specific formulation of priors in our simulation experiments: The support point range for the elasticities is set to 4, so that a rather substantial variation of the estimated elasticities without strong penalties is possible. Two different support centres are considered. In one case they are equal to the true elasticities, in the other case they are shifted upwards by 0.3.

Table 3.1 QP-Model - RMSE of Estimated Gross Margin Elasticity with Different Priors

	Sample size (T)				
Prior information	5	10	20	30	50
'without'	0.187	0.110	0.071	0.055	0.045
'true'	0.158	0.105	0.063	0.055	0.045
'false'	0.163	0.105	0.063	0.055	0.045

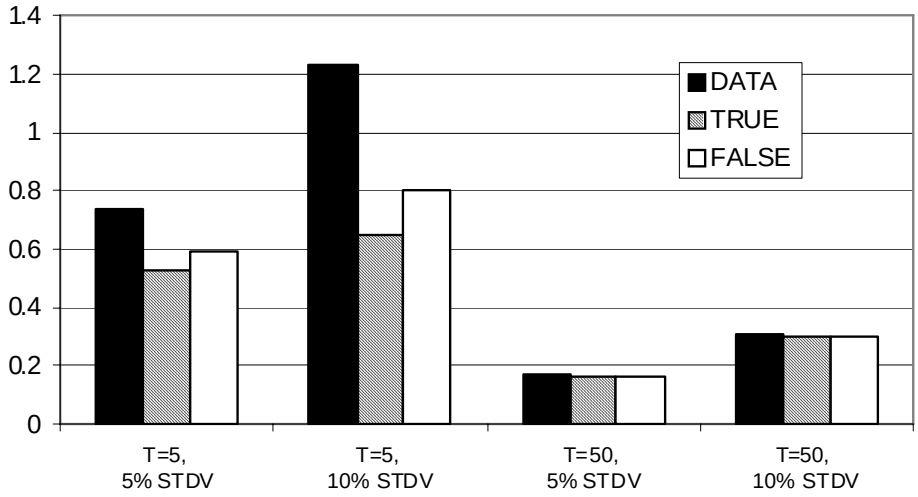
Source: Own calculations. The value of the true gross margin elasticity is 1.03.

Table 3.1 presents the RMSE of the gross margin elasticity of one output at different sample sizes. First we see, that the high variance of parameter estimates displayed in Figure 4.1 is accompanied by a rather stable estimate of elasticities even with little data information. Nevertheless one can infer the general advantage of incorporating the prior information: The estimation error decreases for small sample sizes for both formulations of the priors, although the 'true' prior shows some advantage at the sample size of 5. Beyond a sample size of 20 no differences between the three variants exist and they all approach the true parameters with increasing number of observations.

The impact and usefulness of prior information is certainly also related to the noise in the data generation process. Figure 3.2 shows sums of root mean square errors across all three gross

margin elasticities for two different standard deviations of the error terms (measured in percent of the mean land allocation). It becomes clear that the relative advantage of using priors at small sample sizes increases with the noise in the data generation process. However, for both versions, a sample size of 50 is enough to render the priors almost irrelevant for the quality of the estimates.

Figure 3.2 QP-Model - SRMSE of Estimated Gross Margin Elasticities under Different Priors and Noise Components



Source: Own calculations.

Note, that the inclusion of prior information at small sample sizes can be seen as an intermediate approach between the calibration of the model to exogenous elasticities at some base year value and the estimation of model parameters with sufficient data information. It allows to use at least the little data information available without jeopardising the 'plausibility' of the estimation results.

3.3.4 Model Calibration to a Base Year

With 'calibration', we here mean the approach of fitting model solutions of endogenous variables *exactly* to observed behaviour of one 'base year', i.e. without any deviations on the variables in form of error terms.[29] This is obviously only possible if the number of model parameters to be calibrated is at least equal to the number of independent pieces of information. For our quadratic programming model we have to determine N(1+1/2) = 6 elements of the symmetric matrix **Q** plus N = 3 elements in **d**. These 9 parameters are chosen such that they satisfy only the 3 first

order conditions defined by (3.9). This implies an infinite number of possible solutions to the calibration problem. We should note that in the error free world of calibration, the land constraint (3.10) is either satisfied by 'data construction' (available quantity = sum of all observed land allocations $\mathbf{l}^o$) or does not provide any information at all if it is not binding.

This parameter specification problem is analogous to the one reviewed in the context of PMP in section 2.2.2, with the difference that the shadow price λ and the 'marginal variable cost' $\mathbf{d} + \mathbf{Ql}$ are not determined by the first phase of PMP. There, we already argued that the scarcity of data always requires to incorporate substantial prior information, either directly on the parameters of the model or functions thereof, which is a distinct idea from setting model parameters to 'arbitrary' values. With respect to the *marginal* properties of the programming model, one observation implies absolute ignorance on the data side. All information on how producers react to changing economic conditions must come from prior information, i.e. non-sample information. A natural choice given the availability of other supply studies is prior information on supply elasticities which we will employ below again.

We already know that for the current model with constant yields and gross margins per acre, the employed 'supply elasticities' can be defined as land allocation elasticities with respect to changes in gross margins. The correct full matrix of gross margin elasticities at observed variable quantities is then given by leaving out the 'diag' operator in (3.19) to obtain

$$\mathbf{E} = \frac{\partial \mathbf{l}}{\partial \mathbf{gm}} \odot \left[\frac{\overline{\mathbf{gm}}}{\overline{\mathbf{l}}^o} \right]' = \left(\mathbf{Q}^{-1} - \mathbf{Q}^{-1}\mathbf{u}\left(\mathbf{u}'\mathbf{Q}^{-1}\mathbf{u}\right)^{-1}\mathbf{u}'\mathbf{Q}^{-1} \right) \odot \left[\frac{\overline{\mathbf{gm}}}{\overline{\mathbf{l}}^o} \right]' . \tag{3.23}$$

Before we use this matrix of gross margin elasticities in a GME-D approach we want to first look at two simpler cases, which might be relevant for applications and provide some insight into the rules governing the calibration procedure.

The first case is equivalent to previous practices in PMP model specification which relies on a 'wrong' elasticity definition ignoring the implicit effect of changing activity levels on the shadow price λ (second summand in the outer parentheses in (3.23) vanishes) and sets all off-diagonal elements of $\mathbf{Q}$ a-priori equal to zero. Each single own gross margin elasticities can then simply be calculated as

[29] Generally, one can view calibration as a special case of estimation with negative degrees of freedom, which would include situations with more observations but still more parameters. For an overview on calibration literature, its relation to estimation, and the discussion around it in the economic literature see DAWKINS et.al (2001).

$$\varepsilon_{ii} = q_{ii}^{-1} \cdot \frac{gm_i^o}{l_i^o} \quad \forall i \,. \tag{3.24}$$

Suppose at first, we believed that the true own gross margin elasticities lie somewhere around 3 and we do not have any information on cross price elasticities. Then, equation (3.24) can be used to solve for the parameter q_{ii}. Subsequently each linear parameter d_i must be derived in order to ensure calibration of the model to observed land allocations at observed gross margins. This can be done based on the first order conditions ((3.10) in single equation form)

$$gm_i - \lambda - d_i - q_{ii} l_i = 0 \Leftrightarrow d_i = gm_i - \lambda - q_{ii} l_i \quad \forall i \tag{3.25}$$

only if the shadow price of land λ is set exogenously as well (for example at the value of potentially observed land leasing rates), since otherwise one of the d_i's is not identified. Here we simply set λ equal to the gross margin of the marginal crop as would be implied by the first phase of PMP. Based on observed land allocations and gross margins following from HOWITT's data set (see also the appendix) which are

$$\mathbf{l}^o = \begin{Bmatrix} 1.49 \\ 0.62 \\ 0.54 \end{Bmatrix} \quad \text{and} \quad \mathbf{gm}^o = \begin{Bmatrix} 598.99 \\ 200.07 \\ 406.27 \end{Bmatrix}$$

the following parameter values result:

$$\hat{\mathbf{d}} = \begin{Bmatrix} 199.25 \\ -66.69 \\ 70,77 \end{Bmatrix} \quad \text{and} \quad \hat{\mathbf{Q}} = \begin{Bmatrix} 134.00 & 0 & 0 \\ 0 & 107.57 & 0 \\ 0 & 0 & 250.78 \end{Bmatrix}.$$

Because of the land constraint, the implied (by (3.23)) elasticity matrix $\hat{\mathbf{E}}$ certainly has non-zero cross elasticities. More disturbing is the fact, that it also significantly deviates from the prior information with respect to the diagonal elements due to ignoring the change in the shadow price of land:[30]

$$\hat{\mathbf{E}} = \begin{Bmatrix} 1.921 & -0.449 & -0.391 \\ -3.231 & 1.656 & -1.171 \\ -1.591 & -0.662 & 2.423 \end{Bmatrix}$$

[30] Setting λ to some base year value as has been done for the calibration of parameters does not mean that it stays constant in simulations with the resulting programming model.

This considerable discrepancy from the prior information is neither acceptable nor necessary, because we can simply use - instead of (3.24) - the correct definition of own gross margin elasticities under a diagonal **Q** matrix

$$\varepsilon_{ii} = \left(q_{ii}^{-1} - \left(\sum_{j=1}^{N} 1/q_{jj} \right) \cdot q_{ii}^{-2} \right)^{-1} \cdot \frac{gm_i^o}{l_i^o} \quad \forall i \tag{3.26}$$

for our calibration purposes. Equation (3.26) accounts for the indirect effect of changes in land allocations on the shadow price of land. There even exist a closed form solution for (3.26), however, the formulation is tedious and instead we can numerically solve for the three parameters and afterwards proceed in the same way to obtain the corresponding elements of **d**. The program QPCAL.GMS in section A3.6 of the appendix employs this approach. For convenience we used an appropriately modified version of the GME-D estimation from above, but note that in this case the algorithm simply solves for the unique solution of the parameters at a given λ. The resulting parameter and elasticity estimates are

$$\hat{\mathbf{d}} = \begin{Bmatrix} 247.27 \\ -23.34 \\ 85.60 \end{Bmatrix}, \hat{\mathbf{Q}} = \begin{Bmatrix} 101.78 & 0 & 0 \\ 0 & 37.65 & 0 \\ 0 & 0 & 223.30 \end{Bmatrix} \text{ and } \hat{\mathbf{E}} = \begin{Bmatrix} 3.00 & -0.86 & -0.29 \\ -6.17 & 3.00 & -1.97 \\ -1.19 & -1.08 & 3.00 \end{Bmatrix}$$

which directly reflects the prior information on own gross margin elasticities.

We now turn to the most general calibration approach in the context of our model aiming at the calibration of a full **Q** matrix based on prior information on a complete matrix of own and cross price elasticities defined in (3.23). In this case it is not possible to exactly calibrate to observed land allocations *and* the elasticities, because we have an overidentified system with 9 elasticities plus 3 elements in the land allocation vector (= 12), but only 9 parameters to be calibrated. Therefore, we allow deviations from the prior information on elasticities by implementing a GME-D approach which similar to the estimation approach with prior information above, but does not require any errors on land allocations and uses a full matrix of own gross margin elasticities.

In order to implement the GME-D approach we again define two support points for each element in the 1×9 vector of gross margin elasticities vec(**E**).[31] We reparameterise vec(**E**) as $\mathbf{V}^E\mathbf{w}^E$ where $\mathbf{V}^E$ is a 9×18 matrix of elasticity support points and $\mathbf{w}^E$ the 18×1vector of associated

[31] The operator 'vec' applied to a matrix stacks the columns of the matrix into a column vector.

probabilities. The specific support points are chosen such that the support point centre is equal to the assumed prior expectation

$$\mathbf{E}^{\mathbf{p}} = \begin{bmatrix} 3 & -0.5 & -0.5 \\ -4 & 3 & -1 \\ -2 & -0.5 & 3 \end{bmatrix}$$

and the support range around the centre is equal to 6. The full (degenerated) GME-D formulation also needs to ensure for the positive (semi-) definiteness of **Q** so that the resulting program to exactly calibrate model (3.8) to base year data is given by

$$\max_{\mathbf{w}^{\mathbf{E}},\mathbf{Q},\mathbf{L}} \mathrm{H}\left(\mathbf{w}^{\mathbf{E}}\right) = -\mathbf{w}^{\mathbf{E}}{}' \ln \mathbf{w}^{\mathbf{E}} \tag{3.27}$$

subject to

$$\mathbf{gm} - \boldsymbol{\lambda} - \mathbf{d} - \mathbf{Ql} = \mathbf{0} \tag{3.28}$$

$$\mathbf{Q} = \mathbf{LL}' \quad \text{with } \mathrm{L}_{ij} = 0 \text{ for } j > i \tag{3.29}$$

$$\mathbf{V}^{\mathbf{E}}\mathbf{w}^{\mathbf{E}} = \mathrm{vec}\left(\left(\mathbf{Q}^{-1} - \mathbf{Q}^{-1}\mathbf{u}\left(\mathbf{u}'\mathbf{Q}^{-1}\mathbf{u}\right)^{-1}\mathbf{u}'\mathbf{Q}^{-1}\right) \odot \left[\frac{\overline{\mathbf{gm}}}{\overline{\mathbf{l}}^{\mathrm{o}}}\right]'\right) \tag{3.30}$$

$$\sum_{s=1}^{2} \mathrm{w}_{ijs}^{\mathbf{E}} = 1 \quad \forall i, j \tag{3.31}$$

where w_{ijs} is the probability associated with the support point s of the elasticity ε_{ij}. The calibration results (see QPCAL1.GMS in section A3.6 of the appendix) for the parameters **d** and **Q**, as well as for the resulting elasticities at the base year level are given by

$$\hat{\mathbf{d}} = \begin{Bmatrix} 0.02 \\ -546.83 \\ 182.78 \end{Bmatrix}, \hat{\mathbf{Q}} = \begin{Bmatrix} 197.31 & 192.55 & -26.84 \\ 192.55 & 367.01 & 59.96 \\ -26.84 & 59.96 & 48.58 \end{Bmatrix} \text{ and } \hat{\mathbf{E}} = \begin{Bmatrix} 2.62 & -0.62 & -0.53 \\ -4.44 & 2.13 & -1.32 \\ -2.15 & -0.75 & 2.98 \end{Bmatrix}.$$

The deviations of the elasticities relative to the prior is moderate. This means that the prior elasticity matrix was already relatively consistent with the structure of the model.

There are certainly alternatives to the calibration approach presented here. For example, one could introduce reparameterised error terms on land allocations just as in the estimation exercises presented in the previous section. This would allow to calibrate exactly to the prior elasticities if the support range of the land allocation errors were sufficiently large. Playing with the range of

elasticity and error supports the analyst can adjust to the acceptable level of 'non-calibration' to observed land allocations in trade off with the closeness in matching prior elasticities.

This section shall suffice to show that the calibration of programming models with the GME-D approach can really be described as 'estimation' under negative degrees of freedom. Formally, the entropy programs to estimate model parameters under prior information can be used directly for calibration. Besides parameter identification, the only issue to worry about is that the number of parameters is matched by an at least equal sum of data constraints and pieces of prior information, because only then a unique solution can be obtained. For the next two programming models we will just consider well posed estimation problems and various related Monte Carlo simulations and trust that the reader can straightforwardly apply the methodology to the 'special case' of calibration.

3.4 Input Allocation With Crop Specific Production Functions

3.4.1 Model Overview and Estimation Approach

In this section we want to look at a programming model which allocates variable and fixed inputs to different production activities with a functional representation of crop-specific production technology. The general form of the desired profit maximisation model is given by

$$\max_{x_{ik},b_{ij}} Z = \sum_{i=1}^{N} p_i f_i\left(x_{ik}, b_{ij} \mid \boldsymbol{\theta}_i\right) - \sum_{i=1}^{N}\sum_{k=1}^{K} q_k x_{ik}$$

(3.32) subject to

$$\sum_{i=1}^{N} b_{ij} = b_j \quad \left[\lambda_j\right]$$

where i, j , k are indices of outputs as well as fixed and variable inputs, respectively and $\boldsymbol{\theta}_i$ is a vector of technological parameters. Prices and allocated variable inputs are denoted as q_k and x_{ik}, b_{ij} and b_j represent allocated and available total quantities of the fixed inputs. The transformation of input to output quantities is given by

$$y_i = f_i\left(x_{ik}, b_{ij} \mid \boldsymbol{\theta}_i\right) \quad \forall i \; . \tag{3.33}$$

The first order conditions comprise the resource constraints, the marginal value product conditions of variable inputs, and the shadow price equations of fixed factors:

$$\sum_{i=1}^{N} b_{ij} = b_j \quad \forall j \tag{3.34}$$

$$\frac{\partial Z}{\partial x_{ik}} = p_i \frac{\partial f_i\left(x_{ik}, b_{ij} \mid \boldsymbol{\theta}_i\right)}{\partial x_{ik}} - q_k = 0 \quad \forall i,k, \tag{3.35}$$

$$\frac{\partial Z}{\partial b_{ij}} = p_i \frac{\partial f_i\left(x_{ik}, b_{ij} \mid \boldsymbol{\theta}_i\right)}{\partial b_{ij}} - \lambda_j = 0 \quad \forall i,j. \tag{3.36}$$

To solve this system of first order conditions for the input demand and output supply functions is very cumbersome if not impossible. Instead, we can use equations (3.33) - (3.36) directly as data constraints for estimating the unknowns $\boldsymbol{\theta}_i$ and λ_j. This implies a considerable advantage with respect to the choice of functional form as well as model complexity.[32]

Again, we assume that the data generation process is disturbed by random errors around the endogenous model variables, here not only land allocations, but all input allocations x_{ik} and b_{ij} as well as supply quantities y_i. The corresponding errors e^x_{ikt}, e^b_{ijt}, and e^y_{it} for each observation are reparameterised as

$$\begin{aligned} e^x_{ikt} &= \mathbf{v}^x_{ik}\mathbf{w}^x_{ikt} \quad \forall i,k,t \\ e^b_{ijt} &= \mathbf{v}^b_{ij}\mathbf{w}^b_{ijt} \quad \forall i,j,t \quad \text{and} \\ e^y_{it} &= \mathbf{v}^y_{i}\mathbf{w}^y_{it} \quad \forall i \end{aligned} \tag{3.37}$$

with the (1×2) vectors $\mathbf{v}^x_{ik}$, $\mathbf{v}^b_{ij}$, and $\mathbf{v}^y_i$ representing lower and upper support point and the (2×1) $\mathbf{w}^x_{ikt}$, $\mathbf{w}^b_{ijt}$, and $\mathbf{w}^y_{it}$ their corresponding probabilities for each observation. Adding indices for observations t = 1,...,T to the conditions (3.33) - (3.36) we obtain the complete GME program as

$$\begin{aligned} &\max_{\mathbf{w}^x_{ikt}, \mathbf{w}^b_{ijt}, \mathbf{w}^y_{it}, \boldsymbol{\theta}_i, \lambda_{jt}} H\left(\mathbf{w}^x_{ikt}, \mathbf{w}^b_{ijt}, \mathbf{w}^y_{it}\right) = \\ &-\sum_{i=1}^{N}\left[\sum_{t=1}^{T}\sum_{k=1}^{K}\mathbf{w}^x_{ikt}\,'\ln \mathbf{w}^x_{ikt} + \sum_{t=1}^{T}\sum_{j=1}^{M}\mathbf{w}^b_{ijt}\,'\ln \mathbf{w}^b_{ijt} + \sum_{t=1}^{T}\mathbf{w}^y_{it}\,'\ln \mathbf{w}^y_{it}\right] \end{aligned} \tag{3.38}$$

subject to

$$p_{it}\frac{\partial f_i\left(\left(x^o_{ikt} - \mathbf{v}^x_{ik}\mathbf{w}^x_{ikt}\right), \left(b^o_{ijt} - \mathbf{v}^b_{ij}\mathbf{w}^b_{ijt}\right) \mid \boldsymbol{\theta}_i\right)}{\partial x_{ik}} - q_{kt} = 0 \quad \forall i,k,t \tag{3.39}$$

$$p_{it}\frac{\partial f_i\left(\left(x^o_{ikt} - \mathbf{v}^x_{ik}\mathbf{w}^x_{ikt}\right), \left(b^o_{ijt} - \mathbf{v}^b_{ij}\mathbf{w}^b_{ijt}\right) \mid \boldsymbol{\theta}_i\right)}{\partial b_{ij}} - \lambda_{jt} = 0 \quad \forall i,j,t \tag{3.40}$$

[32] FISCHER et al. (1988: 106f) show how this type of model can also be estimated with a complex iterative approach.

$$(3.41) \qquad \left(y_{it}^{o} - \mathbf{v}_{i}^{y}\mathbf{w}_{it}^{y}\right) = f_{it}\left(\left(x_{ikt}^{o} - \mathbf{v}_{ik}^{x}\mathbf{w}_{ikt}^{x}\right),\left(b_{jt}^{o} - \mathbf{v}_{ij}^{b}\mathbf{w}_{ijt}^{b}\right) | \boldsymbol{\theta}_{i}\right) \quad \forall i,t$$

$$(3.42) \qquad \sum_{i=1}^{N}\left(b_{ijt}^{o} - \mathbf{v}_{ij}^{b}\mathbf{w}_{ijt}^{b}\right) = b_{jt} \quad \forall j,t$$

$$(3.43) \qquad \sum_{s=1}^{S} w_{iks}^{x} = 1 \quad \forall i,k,t; \quad \sum_{s=1}^{S} w_{ijs}^{b} = 1 \quad \forall i,j,t; \quad \sum_{s=1}^{S} w_{its}^{y} = 1 \quad \forall i,t\,.$$

Again, the data constraints have to be satisfied at estimated values of the endogenous variables calculated as the observed values minus the estimated errors.[33]

3.4.2 Monte Carlo Simulations without and with Prior Information on Shadow Prices

Before going to the set up and results of Monte Carlo simulations for this model we want to introduce some prior information also for this model to test its impact on the estimators accuracy. For this purpose we assume – as a variation from the previous model – that we have some information on the mean value of shadow prices of the fixed factors.[34] The GME approach needs to be modified by adding a constraint with the reparameterised mean shadow prices for – here two – fixed factors

$$(3.44) \qquad \mathbf{V}^{\lambda}\mathbf{w}^{\lambda} = \begin{bmatrix} v_{11}^{\lambda} & v_{12}^{\lambda} & 0 & 0 \\ 0 & 0 & v_{21}^{\lambda} & v_{22}^{\lambda} \end{bmatrix} \begin{bmatrix} w_{11}^{\lambda} \\ w_{12}^{\lambda} \\ w_{21}^{\lambda} \\ w_{22}^{\lambda} \end{bmatrix} = \begin{bmatrix} \frac{1}{T}\sum_{t=1}^{T}\lambda_{1t} \\ \frac{1}{T}\sum_{t=1}^{T}\lambda_{2t} \end{bmatrix}.$$

Also, the objective function is extended by the additional probabilities to read

$$(3.45) \qquad \begin{aligned} &\max_{\mathbf{w}_{ikt}^{x},\mathbf{w}_{ijt}^{b},\mathbf{w}_{it}^{y},\mathbf{w}^{\lambda},\boldsymbol{\theta}_{i},\lambda_{jt}} H\left(\mathbf{w}_{ikt}^{x},\mathbf{w}_{ijt}^{b},\mathbf{w}_{it}^{y},\mathbf{w}^{\lambda}\right) = \\ &-\sum_{i=1}^{N}\left[\sum_{t=1}^{T}\sum_{k=1}^{K}\mathbf{w}_{ikt}^{x}\,'\ln\mathbf{w}_{ikt}^{x} + \sum_{t=1}^{T}\sum_{j=1}^{M}\mathbf{w}_{ijt}^{b}\,'\ln\mathbf{w}_{ijt}^{b} + \sum_{t=1}^{T}\mathbf{w}_{it}^{y}\,'\ln\mathbf{w}_{it}^{y}\right] - \sum_{t=1}^{T}\mathbf{w}^{\lambda}\,'\ln\mathbf{w}^{\lambda}\,. \end{aligned}$$

The functional form for the production technology chosen for the Monte Carlo simulations is the 'Constant Elasticity of Substitution' (CES) function, which distinguishes between two variable

[33] The introduction of error terms around the endogenous variables x_{ikt} *and* b_{ijt}, allows to account for the presumed quality of 'observed' input allocations by varying the size of the support range.

[34] The employment of prior information on elasticities for this model is also possible despite the fact that an analytical expression for the elasticities might not be available. One can use discrete approximations based on additional 'artificial' constraints which are simply copies of the data constraints, but with systematically varied exogenous prices and variable 'simulated' output and input quantities. See the appendix, section A3.5 for an illustration of the approach.

(chemicals and capital) and two fixed inputs (land and water).[35] This model structure is analogous to the PMP-CES approach by HOWITT (1995b). However, the current model does not contain any additional non-linear cost terms in the objective function (and the estimation approach certainly does not require any determination of dual values of calibration constraints from the 'phase 1' of PMP). An additional difference is the requirement of decreasing returns to scale in order to allow for positive production levels of all crops.[36]

The data generation process (see also CESSIM.GMS in section A3.6 of the appendix) adds normally distributed error terms to the optimal output and input quantities (with a standard deviation of 10% and 2% from the mean quantities, respectively) to obtain 'observed' allocations, where again the 'incorrectly measured' allocated quantities of the fixed inputs add up exactly to the available and known total quantities. For the simulation we distinguish again between a 'true' and a 'false' prior. The former defines supports for the shadow prices of land and water around the true values at the mean of the observations. The latter is defined by a support centre which is 10% below the true values. The size of supports is chosen to be 40% of the true mean shadow price. This is well above 5 standard deviations of the mean shadow prices across samples so that the support contains the true mean shadow price for both types of priors with almost certainty.

Figure 3.3 shows the absolute bias and the root mean square error as sums over the parameters of all three production functions (SABIAS and SRMSE). The decreasing SRMSE with increasing sample size hints at a consistent estimation approach. The employment of both types of prior information again reduces the SRMSE compared with the pure data case. The reduction is relatively modest compared to the priors on elasticities for the QP-model, but it is still relevant even for T = 100. However, the difference between the 'true' and the 'false' prior is negligible from T = 30 upwards. It is interesting to note, that the bias of the 'true' prior lies above the one for the data. This can happen in the case of a biased estimator and should be seen as an 'accident' resulting from the fact that the prior is applied to an implicit function of parameters and not on the parameters directly.

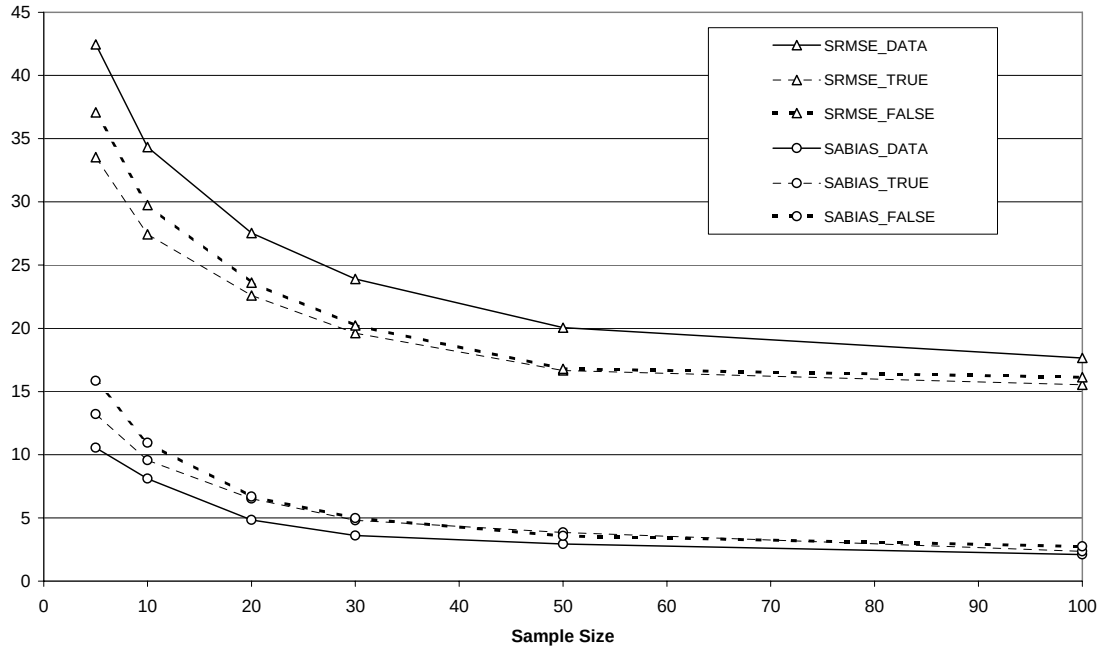
Generally, the prior information for the shadow prices could also be formulated for every observation t instead of the mean of the shadow price, which might better reflect the type of data

[35] For the parameter values of the true model see the appendix. Additional parametric restrictions included during estimation in order to obtain a well defined production function are reported in the appendix as well.

[36] Constant returns to scale (as in HOWITT 1995b) would result in specialisation, since the maximum profit per unit of land in each activity would be independent of the land allocation. Consequently, the number of positive activity levels at the optimum could not be larger than the number of fixed factors as in a linear programming model.

available (e.g. leasing rates for each observation). In this case, however, the number of associated probabilities in the objective function also increases with increasing observations. This may harm the convergence of the estimates to the true parameter values if the empirically unavoidable case occurs that the centres of the shadow price supports are not the true values. Additional simulations not reported here confirmed this hypothesis. This effect, however, could be compensated by including a factor in the objective function which continuously decreases the weight of the prior related probabilities with increasing sample size.[37]

Figure 3.3 SRMSE and SABIAS of Parameter Estimated with Different Prior Information on Shadow Prices of Fixed Resources in the CES Model



Source: Own calculations.

3.5 Allocation of Fixed Inputs with Crop Specific Profit Functions

3.5.1 Model Overview and Estimation Approach

The last example of a programming model keeps the general model structure of the last subsection with respect to assumptions on producer behaviour and crop specific technologies with allocable inputs, but employs duality concepts for the determination of variable output and input quantities. Going back to the case with only one fixed factor, the specification is equivalent to GUYOMARD et al. (1996) and MORO and SCHOKAI (1999), who base their analysis on

[37] The factor γ of the generalised cross entropy objective function (3.7) could, for example, be calculated as (T-0.5)/T.

econometrically estimated systems of supply and explicit land allocation functions. On the one hand, we want to point out the full equivalence of our approach with respect to parameter estimation. On the other hand, we want to illustrate the advantages with respect to flexibility in the choice of functional form as well as the accommodated complexity of the model structure. The desired programming model is given by:

$$\max_{\mathbf{l}} Z = \sum_{i=1}^{N} \pi_i\left(p_i, \mathbf{q}, l_i \mid \boldsymbol{\theta}_i\right)$$

(3.46) subject to

$$\sum_{i=1}^{N} l_i = b \quad [\lambda]$$

where

$$\pi_i\left(p_i, \mathbf{q}, l_i \mid \boldsymbol{\theta}_i\right) = \max_{y_i, \mathbf{x}_i}\left[p_i y_i - \sum_{k=1}^{K} q_k x_{ik} \quad \text{subject to} \quad y_i = f_i\left(x_{ik}, l_i\right)\right] \tag{3.47}$$

is a restricted profit function of crop i for a given land allocation l_i, and $\boldsymbol{\theta}_i$ is now a vector of profit function parameters for product i. Model (3.46) distributes the available land b to the different production activities to maximise overall profit Z, where the profit of the single crops is determined by $\pi_i(p_i,\mathbf{q},l_i|\boldsymbol{\theta}_i)$. Consequently, the optimal land allocation is obtained if the marginal profits of land in each use are equal, i.e. if the first order conditions

$$\frac{\partial \pi_i\left(p_i, \mathbf{q}, l_i \mid \boldsymbol{\theta}_i\right)}{\partial l_i} - \lambda = 0 \tag{3.48}$$

are satisfied. For some functional forms of $\pi_i(.)$ a solution of system (3.48) under additional consideration of the land constraint is possible and results in explicit land allocation equations depending on exogenous model parameters. GUYOMARD et al. (1996) describe the derivation based on normalised quadratic profit functions and estimate a system of land allocation equations and supply functions conditional on land allocations

$$\frac{\partial \pi_i\left(p_i, \mathbf{q}, l_i \mid \boldsymbol{\theta}_i\right)}{\partial p_i} = y_i\left(p_i, \mathbf{q}, l_i \mid \boldsymbol{\theta}_i\right). \tag{3.49}$$

The resulting system is linear, but the regression coefficients have to satisfy non-linear constraints across equations. With our approach, no derivation of land allocation equations is necessary. Instead, the optimality conditions (3.48) are directly used in combination with (3.49) as data constraints of a GME approach analogous to the two cases in the previous subsections. As long as the same statistical model and econometric criterion is employed, the parameter

estimates of this approach must be equal to the ones stemming from the estimation of the behavioural function, because of the mathematical equivalence of the data constraints. This could be confirmed on the basis of a GME and a non-linear least squares approach.

Why would we then want to estimate the model using the optimality conditions and subsequently employ a programming model for simulation purposes? The following points are to be mentioned:

1) The flexibility in choosing the functional form for $\pi_i(p_i,\mathbf{q},l_i)$ is greatly enhanced, because no closed form solution for land allocation functions is necessary.
2) For the same reason, a more complex model structure with more than one fixed factor or political constraints on land allocation (e.g. quotas, base areas...) is not a principal impediment anymore for the econometric estimation of the parameters.
3) The formulation of the resulting simulation model as an explicit optimisation model allows the flexible incorporation of additional relevant constraints on allocation for the simulation horizon without necessarily obstructing the structural validity of the estimation results.

3.5.2 Monte Carlo Simulation Results without Prior Information

Also for this model we performed simulation experiments based on an appropriate GME estimator (see NQSIM.GMS in the appendix). We mirror the approach by GUYOMARD et al. (1996) in the sense that we only employ data on supply quantities and land allocations, disregarding possible observations on allocated input quantities and the related input demand functions as data constraints. Reparameterising the errors of these endogenous variables of the programming model in the same way as for the CES production function model we can formulate the GME program for the estimation of the profit function parameters as

$$\max_{\mathbf{w}^l_{it},\mathbf{w}^y_{it},\boldsymbol{\theta}_i,\lambda_t} H\left(\mathbf{w}^l_{it},\mathbf{w}^y_{it}\right) = -\sum_{i=1}^{N}\sum_{t=1}^{T}\left[\mathbf{w}^l_{it}\,'\ln \mathbf{w}^l_{it} + \mathbf{w}^y_{it}\,'\ln \mathbf{w}^y_{it}\right] \tag{3.50}$$

subject to

$$\frac{\partial \pi_i\left(p_{it},\mathbf{q}_t,\left(l^o_{it}-\mathbf{v}^l_i\mathbf{w}^l_{it}\right)\mid\boldsymbol{\theta}_i\right)}{\partial l_i} - \lambda = 0 \quad \forall i,t, \tag{3.51}$$

$$\frac{\partial \pi_i\left(p_i,\mathbf{q},\left(l^o_{it}-\mathbf{v}^l_i\mathbf{w}^l_{it}\right)\mid\boldsymbol{\theta}_i\right)}{\partial p_i} = \left(y^o_{it}-\mathbf{v}^y_i\mathbf{w}^y_{it}\right) \quad \forall i,t \tag{3.52}$$

$$(3.53) \qquad \sum_{i=1}^{N}\left(b_{it}^{o} - \mathbf{v}_{i}^{l}\mathbf{w}_{it}^{l}\right) = b_{t} \quad \forall t$$

$$(3.54) \qquad \sum_{s=1}^{S} w_{its}^{l} = 1 \quad \forall i,t; \quad \sum_{s=1}^{S} w_{its}^{y} = 1 \quad \forall i,t$$

Again, for different sample sizes, generated optimal supply quantities and land allocations were disturbed by normally distributed errors (with standard deviation of 10% and 2% of the mean variable values) and subsequent estimations without the use of prior information on parameters or functions thereof were executed. The results of the 1000 repetitions are given in table 3.2. The change of the different estimation errors (summed over all estimated parameters of the profit functions) indicates a consistent behaviour of the estimator.

Table 3.2 Profit Function Model – Monte Carlo Results without Prior Information

	Sample size (T)						
Measures	4	5	10	20	30	50	100
SRMSE	2965	2888	1212	570	462	346	253
SABIAS	914	900	417	222	159	102	57
SSTD	2715	2672	1102	516	426	325	242

Source: Own calculations.

The high variance part of the mean squared error one more time suggests a large potential of plausible prior information – for example on elasticities – to improve the estimator's precision for small sample sizes. However, we refrain from a demonstration and focus instead on another issue of empirical relevance:

3.5.3 Estimation with Complementary Slackness Conditions

Constraints on allocation such as the land constraint are frequently of the inequality type and across different observations those might be sometimes binding and sometimes not. As long as the data directly tell us whether a constraint is binding or not for each observation, this is straightforwardly handled by setting the shadow prices, a-priori, to zero for observations with non-binding constraints. But because of the noise in the data generation process, it is conceivable that the measured variable quantities give us the wrong impression. Apparently binding constraints might in fact be not binding for the true quantities and vice versa. In this case, we must allow the estimated or 'fitted' variable values to satisfy the constraints either in equality or

inequality form. In principle, this can be easily accommodated by changing the land constraint (35) to the inequality type

$$(3.55) \qquad \sum_{i=1}^{N}\left(b_{it}^{o}-\mathbf{v}_{i}^{l}\mathbf{w}_{it}^{l}\right)\leq b_{t} \quad \forall t$$

and adding the appropriate complementary slackness condition

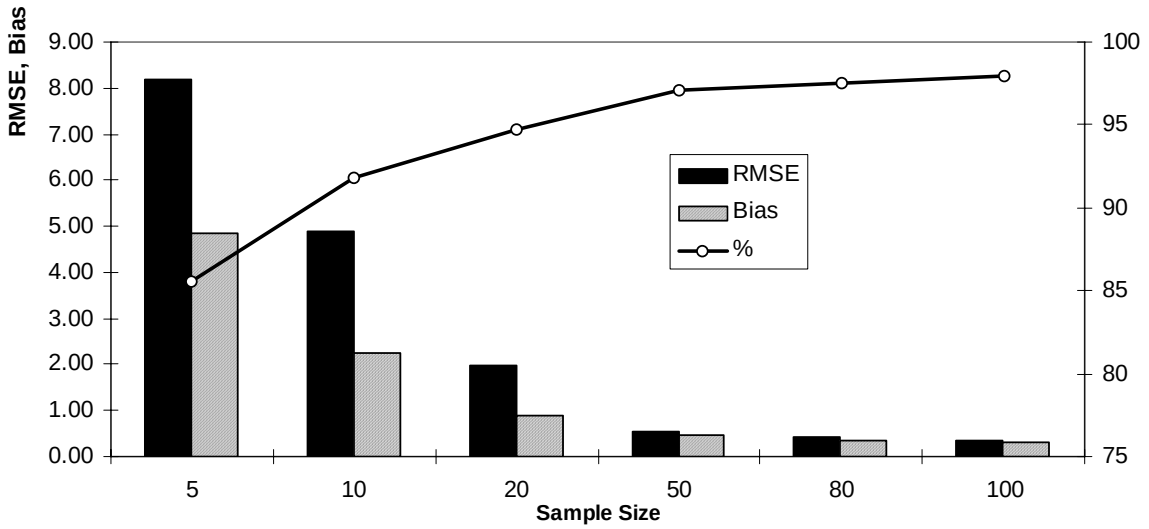
$$(3.56) \qquad \sum_{i=1}^{N}\left(b_{t}-\left(b_{it}^{o}-\mathbf{v}_{i}^{l}\mathbf{w}_{it}^{l}\right)\right)\lambda_{t}=0 \quad \forall t\,,$$

to the GME approach. However, the numerical stability might be hampered with solvers based on gradient methods given the discontinuous nature of λ_t. In order to test this for the relatively simple example above we changed the data generation process for the simulation approach as follows:

First, the available mean level of land was increased in such a way that, on average, about 25% of the optimal solutions of the data generating programming model did not use all the land. Second, we did not enforce that the errors added to optimal land allocations sum to 0 which generally implies a non-zero difference between the sum of 'observed' land allocations and the available total quantity of land. Third, we use equations (3.55) and (3.56) instead of (3.53) in the GME approach. All other specifications of the simulation remained the same. The results are rather promising: As in the above experiments the SRMSE and SABIAS of θ_i indicate a consistent behaviour of the estimation technique. However, to get a better insight of the technique's reflection of the mixed data generation process with non-binding and binding resources, in Figure 3.4 we focus on the finite sample properties of the estimated dual values and on the ability of the approach to correctly identify the status of the constraint.

The RMSE of the dual values is calculated as the square of the difference between the means of the estimated shadow prices across all observations and the means of the true shadow prices for each repetition. Both the bias and the RMSE diminish with increasing sample sizes. To provide further information we include - on the right axis - the percentage of correctly estimated observations concerning binding or non-binding status of the land constraint. It can be concluded that already for small sample sizes the estimation procedure is able to correctly identify binding and non-binding constraints at rates above 85% which is significantly higher than the cut-off value of 75%, i.e. the value obtained by assuming always binding constraints. With increasing sample size the rate almost approaches 100% indicating that the estimates converge to the true data generation process as the amount of data information increases.

Figure 3.4 Bias and RMSE of Land Shadow Prices and Percentage of Correctly Identified Status of the Constraints as Binding or Non-Binding



Source: Own calculations.

3.6 Summary of Chapter 3

This chapter introduces a methodology for the calibration and estimation of programming models based on optimality conditions of the desired model specification. It proves to be an applicable alternative to approaches based on PMP. The method simultaneously allows the specification of more complex models and a more flexible choice of functional form compared to previous estimation approaches of duality based behavioural functions with explicit allocation of fixed factors. The principle procedure and its functionality is demonstrated for three different examples of programming models. Monte Carlo simulations with a maximum entropy criterion indicate consistent behaviour of the estimator. In this context, the potential benefits from prior information on elasticities and shadow prices in situations with small sample sizes as well as the technical implementation was demonstrated. The approach also proved its capability of estimating model parameters across binding and non-binding constraints in the data generation process. Last but not least, the calibration of programming models to base year observations can rely on the same principles and generally allows to employ the same GME formalism as for well posed situations if prior information is used to compensate for the problem of negative degrees of freedom.

Apart from different applications to large, 'real world' profit maximising programming models, many other directions for future research can be identified: extensions of the approach to multi-output production technologies with non-allocable variable factors or to expected utility models

with risk might increase the empirical potential of these types of models by allowing for a higher level of differentiation. From an econometric methodology point of view, the GME approach leaves ample opportunities to improve upon current knowledge although its application is not a necessary requirement for the estimation of programming models in well posed situations: asymptotic properties of the estimator for non-linear models and easily applicable and valid test procedures are still missing. A more systematic investigation with respect to the formulation of prior information and their impact on estimation quality in small sample situations is also desirable.

Appendix to Chapter 3

A3.1 Basic Data Set

The following data set from HOWITT (1995b) provides the differentiation of all presented models with respect to outputs and inputs. Variable quantities are the means of random variables and values of fixed variables for all Monte Carlo simulations.

Table A3.1: Base Year Data on California Agriculture

			Input Allocation			
Crop	Price ($/bu)	Yield (bu/acre)	Land (10^6 acres)	Water (10^6 acre ft)	Capital (Index)	Chemicals (Index)
Cotton	2.924	220	1.49	4.47	3.96	2.64
Wheat	2.98	85	0.62	1.14	1.98	1.32
Rice	7.09	70.1	0.54	3.08	2.94	1.96
Variable Input Prices ($)				10	10	10
Resource Constraints			2.65	8.69		

Source: Based on HOWITT (1995b)

A3.2 Information on Monte Carlo Simulations with QP-Model

True parameter values of **Q** in the Monte Carlo simulations:

$$\mathbf{Q} = \begin{bmatrix} 500 & -20 & -10 \\ -10 & 60 & -2 \\ -10 & -2 & 200 \end{bmatrix}.$$

A3.3 Information on Monte Carlo Simulations with CES-Model

Functional form of production functions and true parameter values:

$$y_i = f\left(x_{ik}, b_{ij} \mid \boldsymbol{\theta}_i\right) = \alpha_i \left(\sum_{k=1}^{2} \beta_{ik} x_{ik}^{\gamma_i} + \sum_{j=3}^{4} \beta_{ij} b_{ij}^{\gamma_i} \right)^{\nu_i/\gamma_i}$$

where

$$\boldsymbol{\theta}_1 = \begin{bmatrix} \alpha_1 \\ \beta_{11} \\ \beta_{12} \\ \beta_{13} \\ \beta_{14} \\ \gamma_1 \\ \upsilon_1 \end{bmatrix} = \begin{bmatrix} 200 \\ 0.2 \\ 0.1 \\ 0.6 \\ 0.1 \\ -0.25 \\ 0.6 \end{bmatrix}; \quad \boldsymbol{\theta}_2 = \begin{bmatrix} \alpha_1 \\ \beta_{11} \\ \beta_{12} \\ \beta_{13} \\ \beta_{14} \\ \gamma_1 \\ \upsilon_1 \end{bmatrix} = \begin{bmatrix} 100 \\ 0.1 \\ 0.1 \\ 0.7 \\ 0.1 \\ -0.25 \\ 0.8 \end{bmatrix}; \quad \boldsymbol{\theta}_3 = \begin{bmatrix} \alpha_1 \\ \beta_{11} \\ \beta_{12} \\ \beta_{13} \\ \beta_{14} \\ \gamma_1 \\ \upsilon_1 \end{bmatrix} = \begin{bmatrix} 50 \\ 0.1 \\ 0.1 \\ 0.5 \\ 0.3 \\ -0.25 \\ 0.8 \end{bmatrix}.$$

Parametric restrictions enforced during estimation:

$$\alpha_i \geq 0; \quad 0 \leq \beta_{ij} \leq 1; \quad \sum_{j=1}^{4} \beta_{ij} = 1 \ ;$$

$$\sigma_i = \frac{1}{1-\gamma_i} \geq 0 \ ; \quad 0 \leq \nu_i \leq 1 \ .$$

A3.4. Information on Monte Carlo Simulations with NQ Profit Function Model

Functional form of the profit functions and true parameter values:

$$\pi_i(p_i, \mathbf{q}, l_i | \boldsymbol{\theta}_i) = \alpha_{0i} + \alpha_{1i} \frac{p_i}{q_2} + \alpha_{2i} \frac{q_1}{q_2} + \alpha_{3i} l_i + 0.5\beta_{1i} \left(\frac{p_i}{q_2} \right)^2 + 0.5\beta_{2i} \left(\frac{q_1}{q_2} \right)^2 + 0.5\beta_{3i} (l_i)^2$$

$$+ \gamma_{1i} \frac{p_i q_1}{q_2^2} + \gamma_{2i} \frac{p_i}{q_2} l_i + \gamma_{3i} \frac{q_1}{q_2} l_i$$

where

$$\boldsymbol{\theta}_1 = \begin{bmatrix} \alpha_{01} \\ \alpha_{11} \\ \alpha_{21} \\ \alpha_{31} \\ \beta_{11} \\ \beta_{21} \\ \beta_{31} \\ \gamma_{11} \\ \gamma_{21} \\ \gamma_{31} \end{bmatrix} = \begin{bmatrix} -67.6571 \\ 115.023 \\ 1.914 \\ 87.836 \\ 24.854 \\ 1.167 \\ -60.321 \\ -9.391 \\ 144.229 \\ -2.883 \end{bmatrix}; \quad \boldsymbol{\theta}_2 = \begin{bmatrix} \alpha_{02} \\ \alpha_{12} \\ \alpha_{22} \\ \alpha_{32} \\ \beta_{12} \\ \beta_{22} \\ \beta_{32} \\ \gamma_{12} \\ \gamma_{22} \\ \gamma_{32} \end{bmatrix} = \begin{bmatrix} -16.2746 \\ -34.130 \\ -0.176 \\ 49.817 \\ 23.552 \\ 0.607 \\ -82.432 \\ -4.232 \\ 135.553 \\ -1.855 \end{bmatrix}; \quad \boldsymbol{\theta}_3 = \begin{bmatrix} \alpha_{03} \\ \alpha_{13} \\ \alpha_{23} \\ \alpha_{33} \\ \beta_{13} \\ \beta_{23} \\ \beta_{33} \\ \gamma_{13} \\ \gamma_{23} \\ \gamma_{33} \end{bmatrix} = \begin{bmatrix} -8.7735 \\ 4.656 \\ -0.650 \\ 27.335 \\ 6.115 \\ 0.882 \\ -53.635 \\ -2.611 \\ 58.284 \\ -2.446 \end{bmatrix}.$$

Parametric restrictions enforced during estimation:

$\beta_{1i} > 0; \quad \beta_{2i} > 0; \quad \gamma_{i2} > 0; \quad \beta_{3i} < 0; \quad \gamma_{i1} < 0.$

Parameters not estimated (only appear in profit function or input demand functions):

$\alpha_{0i}, \alpha_{2i}, \beta_{2i}.$

Parameters not identified relative to shadow price of land (same as in GUYOMARD et al. 1996), where those parameters are part of the composite estimated regression coefficients in the land allocation equations) and therefore fixed at true values:

$\alpha_{31}, \gamma_{31}.$

A3.5 Estimation of CES Model with Prior on Elasticities

The inclusion of prior information on supply elasticities is also possible if a closed form solution on elasticities is not available. The idea is to use discrete approximations of elasticities based upon 'artificial' data constraints that are copies of the regular estimating equations but allow to vary prices systematically and determine their effect on optimal model solutions.[38]

For the case of the CES model, suppose we have prior information on own price elasticities of supply. For every elasticity we need one copy of all first order conditions and the production function to evaluate the change of endogenous model variables – including supply - with respect to a ceteris paribus change of the respective output price at the sample mean (any other variable value for elasticity evaluation is certainly also possible). We index each price regime with p = 1,...,N. Denoting the mean variable values by 'bar', free model variables in the artificial equations with 'tilde', and reparameterising the vector of elasticities equivalent to the presentation of the QP model case as $\boldsymbol{\varepsilon} = \mathbf{V}^{\varepsilon}\mathbf{w}^{\varepsilon}$, the full GME-D approach to estimate the CES model under a prior on own price elasticities is given by

[38] I am indebted to Ron Mittelhammer, who came up with the general idea on how to represent elasticities without closed form solutions in this context.

$$\max_{\mathbf{w}^{x}_{ikt},\mathbf{w}^{b}_{ijt},\mathbf{w}^{y}_{it},\boldsymbol{\theta}_i,\lambda_{jt},\tilde{x}_{ikp},\tilde{b}_{ijp},\tilde{y}_{ip},\tilde{\lambda}_{jp}} H\left(\mathbf{w}^{x}_{ikt},\mathbf{w}^{b}_{ijt},\mathbf{w}^{y}_{it},\mathbf{w}^{\varepsilon}\right) =$$

$$-\sum_{i=1}^{N}\sum_{t=1}^{T}\left[\sum_{k=1}^{K}\mathbf{w}^{x}_{ikt}\,'\ln\mathbf{w}^{x}_{ikt}+\sum_{j=1}^{M}\mathbf{w}^{b}_{ijt}\,'\ln\mathbf{w}^{b}_{ijt}+\mathbf{w}^{y}_{it}\,'\ln\mathbf{w}^{y}_{it}\right]-\mathbf{w}^{\varepsilon}\,'\ln\mathbf{w}^{\varepsilon}$$

subject to

$$p_{it}\frac{\partial f_i\left(\left(x^{o}_{ikt}-\mathbf{v}^{x}_{ik}\mathbf{w}^{x}_{ikt}\right),\left(b^{o}_{ijt}-\mathbf{v}^{b}_{ij}\mathbf{w}^{b}_{ijt}\right)\mid\boldsymbol{\theta}_i\right)}{\partial x_{ik}}-q_{kt}=0\quad\forall i,k,t$$

$$1.01\cdot\overline{p}_{ip}\frac{\partial f_i\left(\left(\tilde{x}_{ikp}\right),\left(\tilde{b}_{ijp}\right)\mid\boldsymbol{\theta}_i\right)}{\partial x_{ik}}-\overline{q}_{kt}=0\quad\forall i,k,p$$

$$p_{it}\frac{\partial f_i\left(\left(x^{o}_{ikt}-\mathbf{v}^{x}_{ik}\mathbf{w}^{x}_{ikt}\right),\left(b^{o}_{ijt}-\mathbf{v}^{b}_{ij}\mathbf{w}^{b}_{ijt}\right)\mid\boldsymbol{\theta}_i\right)}{\partial b_{ij}}-\lambda_{jt}=0\quad\forall i,j,t$$

$$1.01\cdot\overline{p}_{ip}\frac{\partial f_i\left(\left(\tilde{x}_{ikp}\right),\left(\tilde{b}_{ijp}\right)\mid\boldsymbol{\theta}_i\right)}{\partial b_{ij}}-\tilde{\lambda}_{jp}=0\quad\forall i,j,p$$

$$\left(y^{o}_{it}-\mathbf{v}^{y}_{i}\mathbf{w}^{y}_{it}\right)=f_{it}\left(\left(x^{o}_{ikt}-\mathbf{v}^{x}_{ik}\mathbf{w}^{x}_{ikt}\right),\left(b^{o}_{jt}-\mathbf{v}^{b}_{ij}\mathbf{w}^{b}_{ijt}\right)\mid\boldsymbol{\theta}_i\right)\quad\forall i,t$$

$$\tilde{y}_{ip}=f_{ip}\left(\tilde{x}_{ikp},\tilde{b}_{jp}\mid\boldsymbol{\theta}_i\right)\quad\forall i,p$$

$$\sum_{i=1}^{N}\left(b^{o}_{ijt}-\mathbf{v}^{b}_{ij}\mathbf{w}^{b}_{ijt}\right)=b_{jt}\quad\forall i,j,t$$

$$\sum_{i=1}^{N}\left(\tilde{b}_{ijp}\right)=\overline{b}_{j}\quad\forall i,p$$

$$\mathbf{V}^{\varepsilon}\mathbf{w}^{\varepsilon}=\left(\tilde{\mathbf{y}}_{p}-\overline{\mathbf{y}}\right)\odot\left[\frac{\overline{\mathbf{p}}}{\overline{\mathbf{y}}}\right]$$

$$\sum_{s=1}^{S}w^{x}_{ikts}=1;\quad\sum_{s=1}^{S}w^{b}_{ijts}=1;\quad\sum_{s=1}^{S}w^{y}_{its}=1;\quad\sum_{s=1}^{S}w^{\varepsilon}_{is}=1$$

A3.6 GAMS Programs Used for Monte Carlo Simulation and Calibration

QP Cost Function Model - Monte Carlo Simulation (QPSIM.GMS)

```
$OFFLISTING
*-------------------------------------------------------------------------
* QPSIM.GMS QP model estimation and Monte Carlo Simulation without PMP
* Data taken from R.E. Howitt: A calibration Method For Agricultural
* Economic Production ModelsJournal of Agricultural Economics, Vol. 46,
*-------------------------------------------------------------------------

$OFFSYMLIST OFFSYMXREF
 OPTION LIMROW = 0
 OPTION LIMCOL = 0
 option iterlim =2500

*-------------------------------------------------------------------------


 SETS  I       Production processes     /COT Cotton,WHT Wheat,RI Rice/
       IS(I)   Production processes     /COT Cotton,WHT Wheat/
       J       Inputs                   /Land,Water,Capital,Chemical /
       FJ(J)   Fixed inputs             /Land/
       VJ(J)   Variable inputs          /Capital,Chemical,Water/
       G       Observations             /1*500/
       R(G)    Subset of Observation
       K       Support Points           /1,2/
     ;

 ALIAS (I,II,III,IIII);
 ALIAS (J,JJ);

*-------------------------------------------------------------------------
*
*      DATA SET
*
*-------------------------------------------------------------------------

 PARAMETER V(I)   Output price mean
     /COT          2.924
      WHT          2.98
      RI           7.09/;

 PARAMETER YB(I)   Average yields
      /COT        220.0
       WHT         85.0
       RI          70.1/;

 PARAMETER B(FJ,G) Resource availabilities;

 TABLE RC(I,J)     Unit prices of resources

                LAND       WATER      CAPITAL   CHEMICAL
       COT      66.0       25.6         10.0      10.0
       WHT      33.0       25.6         10.0      10.0
       RI       49.0       25.6         10.0      10.0
      ;

 TABLE X(I,J)      Observed resource use

                LAND       WATER       CAPITAL   CHEMICAL
       COT      1.49       4.47         3.960     2.640
       WHT      0.62       1.14         1.980     1.320
       RI       0.54       3.08         2.940     1.960
      ;

*-------------------------------------------------------------------
*
*       Define Model to generate data
*
*-------------------------------------------------------------------

 PARAMETER A(I,J)        Input Coefficients;

       A(I,J) = X(I,J) / X(I,"LAND");
       DISPLAY A;

 TABLE PQ(I,II)    True quadratic parameters of non-linear constraint

                COT       WHT       RI
       COT      500.0    -20.0    -10.0
       WHT      -20.0     60.0     -2.0
```

```
       RI        -10.0     -2.0    200.0;

 TABLE PL(I,II)    Lower Triangular Cholesky Matrix of PQ

                 COT            WHT           RI

       COT       22.360680     0.0            0.0
       WHT      -0.89442719    7.6941536      0.0
       RI       -0.44721360   -0.31192515    14.131621;

 TABLE QI(I,II)    Inverse of Q

                 COT              WHT              RI

       COT    0.0020293680    0.00068006496    0.00010826905
       WHT   0.00068006496    0.016900122      0.00020300447
       RI    0.00010826905    0.00020300447    0.0050074435 ;

 PARAMETER PD(I)    True linear parameters
      /COT        0.0
       WHT        0.0
       RI         0.0/;

 PARAMETER  GMARG(I)         Gross Margins;

       GMARG(I) = YB(I) * V(I) - SUM(VJ,A(I,VJ) * RC(I,VJ));

 PARAMETER  GM(G,I)          Gross Margins in Simulation

 VARIABLES  GL(G,I)          Land allocated to crop I
            GPROFIT          Total profit
           ;

 POSITIVE VARIABLES GL;

 EQUATIONS
            GRES(G,FJ)       Fixed resources
            GPROF            Profit definition
          ;

*
* ---- declare equations -----------------------------------------
*


 GRES(R,FJ)..  SUM(I, A(I,FJ) * GL(R,I))  =L= B(FJ,R);

 GPROF..           GPROFIT     =E=

*                         *** gross margins times land allocated
                         SUM((R,I),   GM(R,I) *  GL(R,I))
*                         *** non-linear cost
                               - SUM((R,I),PD(I)*GL(R,I))
                               - 0.5*SUM((R,I,II),GL(R,I)*PQ(I,II)*GL(R,II))
                        ;

 GL.LO(G,I) = 0.0;

 MODEL LEONGEN /GRES,GPROF/;

* LEONGEN.Optfile = 3;


*---------------------------------------------------------------------
*
*       Define Model for Entropy-Estimation
*
*---------------------------------------------------------------------

 PARAMETER
             LOBS(G,I)       observed land allocation
             SPRERR          support spread of errors (sigma-factor) /0.1/
             SUPERR(G,I,K)   supports errors
             SPRELA          spread gross margin elasticities /2/
             CENELA(I)       center elasticity supports /COT 1.030, WHT
                                                                 0.729, RI 1.290/
             SUPELA(I,K)     supports elasticities
             MEANGM(I)       mean of gross margins
             MEANL(I)        mean of land allocations
             IDENTITY(I,II)  identity matrix
          ;

 SUPERR(G,I,'1') = 0 - SPRERR;
 SUPERR(G,I,'2') = 0 + SPRERR;
* SUPELA(I,'1') = CENELA(I) - SPRELA;
* SUPELA(I,'2') = CENELA(I) + SPRELA;
 SUPELA(I,'1') = CENELA(I) + 0.3 - SPRELA;
```

```
 SUPELA(I,'2') = CENELA(I) + 0.3 + SPRELA;
 IDENTITY(I,II) = 0;
 IDENTITY(I,I)  = 1;

 VARIABLES ENTROPY
           D(I)           linear parameters of non-linear constraint
           Q(I,II)        quadratic parameters of nonlinear constraint
           QINV(I,II)     inverse of Q
           L(I,II)        lower triangular cholesky matrix of Q
           ERROR(G,I)     errors in land allocation
           ELA(I,II)      gross margin elasticities
           W(G,I,K)       probabilities error support
           PELA(I,K)      probabilities elasticity supports
           LAMBDA(G,FJ)   shadow prices
          ;

 POSITIVE VARIABLES W,PELA,LAMBDA
          ;

 EQUATIONS OBJENT         Objective Function
           FOC(G,I)       First Order Conditions
           RES(G,FJ)      Resource Constraint
           CSRES(G,FJ)    Complementary Slackness Resource Constraints
           EXPERR(G,I)    Error = sum over prob * supports
           EXPELA(I)      elasticity  = sum over prob * supports
           ELAEQ(I,II)    elasticity definition
           WSUM(G,I)      sum of prob = 1
           PELASUM(I)     sum of prob = 1
           QINVEQ(I,II)   calculation of Q inverse
           CHOL(I,II)     Cholesky decomposition of Q
          ;

*
* ---- declare equations -----------------------------------------
*

 OBJENT..     - (SUM((R,I,K),W(R,I,K)*LOG(W(R,I,K))))
              - (SUM((I,K),PELA(I,K) *LOG(PELA(I,K))))
                 =E= ENTROPY;


 FOC(R,I)..  GM(R,I) - SUM(FJ,A(I,FJ) * LAMBDA(R,FJ))
             - D(I) - SUM(II,Q(I,II) * (LOBS(R,II) - ERROR(R,II)))
             =E= 0;

 RES(R,FJ)..  SUM(I, A(I,FJ) * (LOBS(R,I) - ERROR(R,I)))  =E= B(FJ,R);

 EXPERR(R,I)..  ERROR(R,I) =E= SUM(K,W(R,I,K)*SUPERR(R,I,K));

 EXPELA(I)..  ELA(I,I) =E= SUM(K,PELA(I,K)*SUPELA(I,K));

 ELAEQ(I,II)..  ELA(I,II) =E= (QINV(I,II) -
                                ((1/SUM((III,IIII),QINV(III,IIII))) *
                              SUM(III,QINV(I,III))*SUM(III,QINV(III,II))))
                              * (MEANGM(II)/MEANL(I));

 QINVEQ(I,II)..    SUM(III,Q(I,III) * QINV(III,II))
               =E= IDENTITY(I,II);

 WSUM(R,I)..  SUM(K,W(R,I,K)) =E= 1;

 PELASUM(I)..  SUM(K,PELA(I,K)) =E= 1;

 CHOL(I,II)..  SUM(III,L(I,III) * L(II,III)) =E= Q(I,II);

 MODEL TOM /OBJENT,FOC,RES,EXPERR,EXPELA,ELAEQ,QINVEQ,WSUM,PELASUM,CHOL/;

* TOM.Optfile = 3;

* ---------------------------------------------------------------
*
* ---- define simulation model ----------------------------------
*
* ---------------------------------------------------------------

 PARAMETERS  PARD(I)             Estim. D-parameters
             PARQ(I,II)          Estim. Q parameters
            ;

 VARIABLES  XN(I)         Resource allocation
            TPROFIT       Total profit
           ;

 POSITIVE VARIABLES XN;

 EQUATIONS
            RESOURCEN(FJ)     Fixed resources
```

```
           PROFIT            Profit definition
         ;

 RESOURCEN(FJ)..  SUM(I, A(I,FJ) * XN(I) )  =L= 2.65;

 PROFIT..           TPROFIT     =E=
*                        *** revenue
                        SUM(I,   MEANGM(I) *  XN(I))

                      - (SUM(I,PARD(I)*XN(I)) +
                      - 0.5*SUM((I,II),XN(I)*PARQ(I,II)*XN(II)))
                         ;

 MODEL CALMOD /RESOURCEN,PROFIT/;

* CALMOD.Optfile = 3;


*--- Simulation Steering -----------------------------------------------

 SET GG / 1*5/;
 PARAMETER NOBS(GG) / 1 5, 2 10, 3 20, 4 30,5 50 /;

 SCALAR NSMP /1000/;

 PARAMETERS  ELAS
             MEANXN
             SMPERR(G,I)         Sampling Errors
             MEANQ(GG,I,II)      Mean of Q estimates
             MEANELA(GG,I)       Mean of elasticity estimates
             MEANLAM(GG,G,FJ)    Mean of Lambda estimates
             BIASQ(GG,I,II)      Bias of Q estimates
             BIASELA(GG,I)       Bias of elasticity estimates
             BIASLAM(GG,G,FJ)    Bias of Lambda estimates
             SABIAS(GG)          Sum of absolute bias of Q
             SQBIAS(GG)          Sum of squared bias of Q
             VARQ(GG,I,II)       Variance Q Estimates
             VARELA(GG,I)        Variance elasticity estimates
             VARLAM(GG,G,FJ)     Variance Lambda Estimates
             SVARQ(GG)           sum of variances of Q
             SSTDQ(GG)           sum of standard deviations of Q
             MSEQ(GG,I,II)       MSE Q Estimates
             MSEELA(GG,I)        MSE elasticity Estimates
             MSELAM(GG,G,FJ)     MSE Lambda Estimates
             SMSEQ(GG)           sum of mean square error of Q
             SRMSE(GG)           sum of root mean square error of Q
             SNRMSE(GG)          sum of normalized root mean square error of Q
             SRMSEELA(GG)        sum of root mean square error of elasticities
             COUNT(GG)           Anzahl Datengenerierungen
             NINFES(GG)          Anzahl der Infeasible Simulationen
             OBS(G)              Observations
             VG(G,I)             Randomized Prices
           ;

*
* ---- sample size loop -------------------------------------------------
*

 LOOP(GG,

     R(G)  = No;
     R(G) $ (ORD(G) LE NOBS(GG)) = YES;

*
* ---- monte carlo loop setup  ------------------------------------------
*
     MEANQ(GG,I,II)    = 0;
     MEANELA(GG,I)     = 0;
     MEANLAM(GG,R,FJ)  = 0;
     VARQ(GG,I,II)     = 0;
     VARELA(GG,I)      = 0;
     VARLAM(GG,R,FJ)   = 0;
     COUNT(GG) = 1;
     NINFES(GG) = 0;

OPTION SOLPRINT = off;

     WHILE(COUNT(GG) LE NSMP,

         VG(R,I) = V(I) + NORMAL(0,0.1 * V(I));
         GM(R,I) = YB(I) * VG(R,I) - SUM(VJ,A(I,VJ) * RC(I,VJ));
         B(FJ,R) = 2.65 + NORMAL(0,0.1);

         SOLVE LEONGEN USING NLP MAXIMIZING GPROFIT;

         MEANL(I) = SUM(R,GL.L(R,I))/CARD(R);
         MEANGM(I) = SUM(R,GM(R,I))/CARD(R);
```

```
*
* ---- set bounds and starting values ------------------------------------------
*

         Q.LO(I,II) = -200;
         Q.LO(I,I) = 0.0001;
         Q.UP(I,II) = 1000;

         QINV.LO(I,II) = -200;
         QINV.UP(I,II) = 200;

         D.LO(I) = -500;
         D.UP(I) = 500;

         W.LO(G,I,K) = 0.00001;
         W.UP(G,I,K) = 1;

         PELA.LO(I,K) = 0.00001;
         PELA.UP(I,K) = 1;

         ELA.LO(I,I) = SUPELA(I,'1');
         ELA.UP(I,I) = SUPELA(I,'2');

         ERROR.LO(G,I) = -SPRERR;
         ERROR.UP(G,I) = SPRERR;

         L.LO(I,II) = -100;
         L.UP(I,II) = 100;

         LAMBDA.LO(G,FJ) = 0;
         LAMBDA.UP(G,FJ) = 100*GRES.M(G,FJ);

         SMPERR(R,I) = NORMAL(0,0.02);
         SMPERR(R,'COT') = -(SMPERR(R,'WHT') + SMPERR(R,'RI'));
         LOBS(R,I) = GL.L(R,I) + SMPERR(R,I);
         Q.L(I,II) = PQ(I,II);
         QINV.L(I,II) = QI(I,II);
         D.FX(I) = PD(I);
         W.L(R,I,'1') = (SMPERR(R,I)-SUPERR(R,I,'2'))/(SUPERR(R,I,'1')-SUPERR(R,I,'2'));
         W.L(R,I,'2') = 1-W.L(R,I,'1');
         ELA.L(I,II) = (QINV.L(I,II) - ((1/SUM((III,IIII),QINV.L(III,IIII))) *
                           SUM(III,QINV.L(I,III))*SUM(III,QINV.L(III,II))))
                           * (MEANGM(II)/MEANL(I));
         PELA.L(I,'1') = (ELA.L(I,I)-SUPELA(I,'2'))/(SUPELA(I,'1')-SUPELA(I,'2'));
         PELA.L(I,'2') = 1-PELA.L(I,'1');
         ERROR.L(R,I) = SMPERR(R,I);
         L.L(I,II) = PL(I,II);
         L.FX(I,II)$(ord(I) lt ord(II)) = 0;
         LAMBDA.L(R,FJ) = GRES.M(R,FJ);
         SOLVE TOM USING NLP MAXIMIZING ENTROPY;

         IF((TOM.modelstat LE 2),
             MEANQ(GG,I,II) = MEANQ(GG,I,II) + Q.L(I,II);
             MEANLAM(GG,R,FJ) = MEANLAM(GG,R,FJ) + LAMBDA.L(R,FJ);
             MEANELA(GG,I) = MEANELA(GG,I) + ELA.L(I,I);
             VARQ(GG,I,II) = VARQ(GG,I,II) + (Q.L(I,II))*(Q.L(I,II));
             VARELA(GG,I) = VARELA(GG,I) + (ELA.L(I,I) * ELA.L(I,I));
             VARLAM(GG,R,FJ) = VARLAM(GG,R,FJ) + (LAMBDA.L(R,FJ) * LAMBDA.L(R,FJ));
             COUNT(GG) = COUNT(GG) + 1;
         ELSE
             NINFES(GG) = NINFES(GG) + 1;
           );

         );

      MEANQ(GG,I,II) = MEANQ(GG,I,II)/NSMP;
      MEANELA(GG,I) = MEANELA(GG,I)/NSMP;
      MEANLAM(GG,R,FJ) = MEANLAM(GG,R,FJ)/NSMP;

      BIASQ(GG,I,II) = MEANQ(GG,I,II)-PQ(I,II);
      BIASELA(GG,I) = MEANELA(GG,I)-CENELA(I);
      BIASLAM(GG,R,FJ) = MEANLAM(GG,R,FJ)-GRES.M(R,FJ);

      VARQ(GG,I,II) = VARQ(GG,I,II)/NSMP - MEANQ(GG,I,II)*MEANQ(GG,I,II);
      VARELA(GG,I) = VARELA(GG,I)/NSMP - MEANELA(GG,I)*MEANELA(GG,I);
      VARLAM(GG,R,FJ) = VARLAM(GG,R,FJ)/NSMP - MEANLAM(GG,R,FJ)*MEANLAM(GG,R,FJ);

      MSEQ(GG,I,II) = VARQ(GG,I,II) + (MEANQ(GG,I,II)-PQ(I,II))*(MEANQ(GG,I,II)-PQ(I,II));
      MSEELA(GG,I) = VARELA(GG,I) + (MEANELA(GG,I)-CENELA(I))*(MEANELA(GG,I)-CENELA(I));
      MSELAM(GG,R,FJ) = VARLAM(GG,R,FJ) + (MEANLAM(GG,R,FJ)-GRES.M(R,FJ))*(MEANLAM(GG,R,FJ)-
                        GRES.M(R,FJ));

      SABIAS(GG)    = SUM((I,II),ABS(BIASQ(GG,I,II)));
      SQBIAS(GG)    = SUM((I,II),SQR(BIASQ(GG,I,II)));
      SVARQ(GG)     = SUM((I,II),VARQ(GG,I,II));
      SSTDQ(GG)     = SUM((I,II),SQRT(VARQ(GG,I,II)));
      SMSEQ(GG)     = SUM((I,II),MSEQ(GG,I,II));
```

```
     SRMSE(GG)      = SUM((I,II),SQRT(MSEQ(GG,I,II)));
     SNRMSE(GG)     = SUM((I,II),SQRT(MSEQ(GG,I,II))/ABS(PQ(I,II)));
     SRMSEELA(GG)   = SUM((I),SQRT(MSEELA(GG,I)));


*
* ----- Solve Simulation model at data mean --------------------------
*
     PARD(I)     =  0;
     PARQ(I,II)  =  MEANQ(GG,I,II);

     XN.LO(I) = 0.0;
     XN.UP(I) = 100*SUM(R,GL.L(R,I))/CARD(R);

     SOLVE CALMOD USING NLP MAXIMIZING TPROFIT;


*---------------------------------------------------------------
*
*       Calculation of GM-elasticities at Mean
*
*---------------------------------------------------------------

     MEANXN(I) = XN.L(I);

     LOOP(III,
*
* --- increase by price by 1 percent
*
         MEANGM(III) = MEANGM(III) * 1.01;

         SOLVE CALMOD USING NLP MAXIMIZING TPROFIT;
*
* --- percentage change in land allocation
*
         ELAS(GG,I,III) = (XN.L(I) - MEANXN(I))/MEANXN(I) * 100.;

         MEANGM(III) = MEANGM(III) / 1.01;
       );
     );

 DISPLAY COUNT,NINFES,NSMP;
 DISPLAY MEANQ,BIASQ,VARQ,MSEQ,MEANLAM,BIASLAM,VARLAM,MSELAM,MEANELA,BIASELA,VARELA,MSEELA;
 DISPLAY ELAS;
 DISPLAY SABIAS,SQBIAS,SVARQ,SSTDQ,SMSEQ,SRMSE,SNRMSE,SRMSEELA;
```

*QP Cost Function Model - Calibration Based on Diagonal **Q** and Diagonal Elasticity Prior (QPCAL.GMS)*

```
*-------------------------------------------------------------------------
*** QPCAL.GMS ***
*
* Calibration of quadratic programming model to observed base
* year quantities. Employment of prior information on own-gross margin elas-
* ticities
*
* Data taken from R.E. Howitt: A Calibration Method For Agricultural
* Economic Production ModelsJournal of Agricultural Economics, Vol. 46, No.2,
* May 1995, pp 147-160
*-------------------------------------------------------------------------

$TITLE LEONTIEF CALIBRATION
$OFFLISTING
$OFFSYMLIST OFFSYMXREF

 OPTION LIMROW = 0
 OPTION LIMCOL = 0
 option iterlim =2500

*-------------------------------------------------------------------------

 SETS  I      Production processes     /COT Cotton,WHT Wheat,RI Rice/
       J      Inputs                   /Land,Water,Capital,Chemical /
       FJ(J)  Fixed inputs             /Land/
       VJ(J)  Variable inputs          /Capital,Chemical,Water/
       K      Support Points           /1,2/
     ;

 ALIAS (I,II,III);
 ALIAS (J,JJ);

*-------------------------------------------------------------------------
*
*      DATA SET
*
*-------------------------------------------------------------------------

 PARAMETER V(I)   observed output prices
     /COT          2.924
      WHT          2.98
      RI           7.09/;

 PARAMETER YB(I)   Average yields
      /COT        220.0
       WHT         85.0
       RI          70.1/;

 PARAMETER B(FJ)   Resource constraints
      /LAND        2.65/;

 TABLE RC(I,VJ)    Unit prices of resources

                  CAPITAL   CHEMICAL
       COT          10.0      10.0
       WHT          10.0      10.0
       RI           10.0      10.0
      ;

 TABLE X(I,J)      Observed resource use

                LAND       WATER       CAPITAL   CHEMICAL
       COT      1.49       4.47         3.960     2.640
       WHT      0.62       1.14         1.980     1.320
       RI       0.54       3.08         2.940     1.960
      ;

 PARAMETER A(I,J)        Input Coefficients;

       A(I,J) = X(I,J) / X(I,"LAND");

 PARAMETER GM(I)      Gross Margins;

       GM(I) = YB(I) * V(I) - SUM(VJ,A(I,VJ) * RC(I,VJ));

 DISPLAY GM;

*-------------------------------------------------------------------
*
*       Define Model for Entropy-Estimation
*
```

```
*---------------------------------------------------------------------

 PARAMETER
             CENEL          support center epsilon /3/
             SPREL          support spread epsilon /3/
             SUPEL(I,K)     support points epsilon
          ;

 SUPEL(I,'1') = CENEL - SPREL;
 SUPEL(I,'2') = CENEL + SPREL;

 VARIABLES ENTROPY
           D(I)           linear parameters of non-linear cost
           Q(I,II)        quadratic parameter matrix of nonlinear cost
           PELAS(I,K)     probabilities epsilon
           LAMBDA(FJ)     shadow price land
          ;

 POSITIVE VARIABLES PELAS, LAMBDA
          ;

 EQUATIONS OBJENT         objective function
           FOC(I)         first order conditions
           RES(FJ)        land constraint
           ELAS(I)        elasticity calculation
           ELASUM(I)      sum of prob = 1
          ;

*
* ---- entropy model equations -------------------------------------------
*

 OBJENT..          -(SUM((I,K),PELAS(I,K)*LOG(PELAS(I,K))))
                =E= ENTROPY;

 FOC(I)..          GM(I) - SUM(FJ,A(I,FJ) * LAMBDA(FJ))
                   - D(I) - SUM(II,Q(I,II) * X(II,'LAND'))
                =E= 0;

 RES(FJ)..         SUM(I, A(I,FJ) * X(I,'LAND'))
                =E= B(FJ);

 ELAS(I)..         SUM(K,PELAS(I,K)*SUPEL(I,K))
                =E= (1/Q(I,I)) * (GM(I)/X(I,'LAND'));

 ELASUM(I)..       SUM(K,PELAS(I,K)) =E= 1;

*
* ---- set bounds and starting values ------------------------------------
*

 D.LO(I)    = -1000;
 D.UP(I)    = 1000;

 Q.LO(I,I)  = 0.001;
 Q.UP(I,I)  = 1000;
 Q.FX(I,II)$(ord(I) ne ord(II)) = 0;
 Q.L(I,I) = (1/CENEL) * (GM(I)/X(I,'LAND'));

 PELAS.LO(I,K) = 0.00001;
 PELAS.UP(I,K) = 1;
 PELAS.L(I,K)  = 0.5;

 ELAS.LO(I) = SUPEL(I,'1');
 ELAS.UP(I) = SUPEL(I,'2');
 ELAS.L(I)  = CENEL;

 LAMBDA.FX('LAND')  = SMIN(I,GM(I));

 MODEL ENTRO /OBJENT,FOC,RES,ELAS,ELASUM/;

 OPTION NLP=CONOPT2;
 ENTRO.Optfile = 2;
 SOLVE ENTRO USING NLP MAXIMIZING ENTROPY;

 PARAMETER ELASTEST;
 ELASTEST(I) = (1/Q.L(I,I)) * (GM(I)/X(I,'LAND'));
 DISPLAY ELASTEST;

*---------------------------------------------------------------------
*
*       Define Calibrated model with non-linear objective function
*
*---------------------------------------------------------------------

 PARAMETERS  PARD(I)       Calibrated D-parameters
             PARQ(I,II)    Calibrated Q parameters
```

```
          ;

 PARD(I)     =  D.L(I);
 PARQ(I,II)  =  Q.L(I,II);

 VARIABLES  XCAL(I,FJ)     land allocation
            PROFIT         profit

 POSITIVE VARIABLES XCAL;

 EQUATIONS
            RESOURCEN(FJ)  Fixed resources
            PROFEQ         Profit definition
          ;

*
* ----  equations of calibrated model ----------------------------
*

 PROFEQ..            PROFIT
                =E= SUM((I),   GM(I) * XCAL(I,'LAND'))
                    - (SUM((I),PARD(I) * XCAL(I,'LAND')
                    + SUM(II,0.5 * XCAL(I,'LAND') * PARQ(I,II) * XCAL(II,'LAND'))));

 RESOURCEN(FJ)..    SUM(I, XCAL(I,FJ) )  =L= B(FJ);

 XCAL.LO(I,FJ) = 0.0;
 XCAL.UP(I,FJ) = 100*X(I,FJ);

 MODEL LEONCAL /PROFEQ,RESOURCEN/;

 OPTION NLP=CONOPT2;
 SOLVE LEONCAL USING NLP MAXIMIZING PROFIT;
 LEONCAL.Optfile = 2;

*---------------------------------------------------------------------
*
*       Check if model calibrates
*
*---------------------------------------------------------------------

    PARAMETER    PERDIF(I,FJ)  Percent difference in input allocation;

    PERDIF(I,FJ) = (XCAL.L(I,FJ) - X(I,FJ)) * 100 / X(I,FJ) ;

    DISPLAY PERDIF ;


*---------------------------------------------------------------------
*
*       Calculation of Elasticities
*
*---------------------------------------------------------------------

  PARAMETER ELASCAL;

  LOOP(III,
*
* --- increase of gross margin by 1 percent
*
     GM(III) = GM(III) * 1.01;
     SOLVE LEONCAL USING NLP MAXIMIZING PROFIT;
*
* --- percentage change in land allocation
*
     ELASCAL(I,FJ,III) = (XCAL.L(I,FJ)  - X(I,FJ))/X(I,FJ) * 100.;
*
*
     GM(III) = GM(III) / 1.01;
  );

  DISPLAY ELASCAL;
```

*QP Cost Function Model - Calibration Based on Full **Q** and Full Elasticity Prior (QPCAL1.GMS)*

```
*------------------------------------------------------------------------------
*** QPCAL1.GMS ***
*
* GME-D calibration of quadratic programming model to observed
* base year quantities. Employment of prior information on full matrix of
* gross margin elasticities.
*
* Data taken from R.E. Howitt: A Calibration Method For Agricultural
* Economic Production ModelsJournal of Agricultural Economics, Vol. 46, No.2,
* May 1995, pp 147-160
*------------------------------------------------------------------------------

$TITLE LEONTIEF CALIBRATION
$OFFLISTING
$OFFSYMLIST OFFSYMXREF

 OPTION LIMROW = 0
 OPTION LIMCOL = 0
 option iterlim =2500

*------------------------------------------------------------------------------

 SETS  I      Production processes     /COT Cotton,WHT Wheat,RI Rice/
       J      Inputs                   /Land,Water,Capital,Chemical /
       FJ(J)  Fixed inputs             /Land/
       VJ(J)  Variable inputs          /Capital,Chemical,Water/
       K      Support Points           /1,2/
     ;

 ALIAS (I,II,III,IIII);
 ALIAS (J,JJ);

*------------------------------------------------------------------------------
*
*      DATA SET
*
*------------------------------------------------------------------------------

 PARAMETER V(I)   observed output prices
     /COT          2.924
      WHT          2.98
      RI           7.09/;

 PARAMETER YB(I)   Average yields
      /COT        220.0
       WHT         85.0
       RI          70.1/;

 PARAMETER B(FJ)   Resource constraints
      /LAND        2.65/;

 TABLE RC(I,VJ)    Unit prices of resources

                  CAPITAL   CHEMICAL
       COT          10.0      10.0
       WHT          10.0      10.0
       RI           10.0      10.0
      ;

 TABLE X(I,J)      Observed resource use

                 LAND       WATER       CAPITAL   CHEMICAL
       COT       1.49       4.47         3.960     2.640
       WHT       0.62       1.14         1.980     1.320
       RI        0.54       3.08         2.940     1.960
      ;

 PARAMETER A(I,J)        Input Coefficients;

       A(I,J) = X(I,J) / X(I,"LAND");

 PARAMETER GM(I)      Gross Margins;

       GM(I) = YB(I) * V(I) - SUM(VJ,A(I,VJ) * RC(I,VJ));

 DISPLAY GM;

*------------------------------------------------------------------
*
*       Define Model for Entropy-Estimation
*
```

```
*---------------------------------------------------------------------

 PARAMETER
             SPREL          support spread epsilon /3/
             SUPEL(I,II,K)  support points epsilon
             IDENTITY(I,II) identity matrix
          ;

 IDENTITY(I,I) = 1;

 TABLE CENEL(I,II)      prior expectations of elasticities

                 COT       WHT       RI
       COT        3.       -0.5      -0.5
       WHT       -4.        3.       -1.
       RI        -2.       -0.5       3.
      ;

 TABLE SVQ(I,II)      starting values for Q (inv(cenel * li over gmj))

                 COT              WHT              RI
       COT   0.0014582147    0.00012492404    0.00012434835
       WHT   0.0071904358    0.0017097105     0.00093068920
       RI    0.0020234632    0.00026311812    0.00064166871
      ;

 TABLE SVQI(I,II)      starting values for QINV

                 COT              WHT              RI
       COT   0.0074626339    -0.0041324455    -0.0018048039
       WHT  -0.0041324455     0.0092965513    -0.0014833020
       RI   -0.0018048039    -0.0014833020     0.0039875138
      ;

 TABLE SVL(I,II)      starting values for L (cholesky decomp. of SVQ)

                 COT              WHT              RI
       COT   0.038186577       0.00000000       0.00000000
       WHT   0.0032714122      0.041219029      0.00000000
       RI    0.0032563366      0.022320671      0.011526172
      ;

 SUPEL(I,II,'1') = CENEL(I,II) - SPREL;
 SUPEL(I,II,'2') = CENEL(I,II) + SPREL;

 VARIABLES ENTROPY
           D(I)            linear parameters of non-linear cost
           Q(I,II)         quadratic parameter matrix of non-linear cost
           QINV(I,II)      inverse of Q
           L(I,II)         lower triangular cholesky matrix of Q
           PELAS(I,II,K)   probabilities epsilon
           LAMBDA(FJ)      shadow price land
          ;

 POSITIVE VARIABLES PELAS, LAMBDA
          ;

 EQUATIONS OBJENT          objective function
           FOC(I)          first order conditions
           RES(FJ)         land constraint
           ELAS(I,II)      elasticity calculation
           ELASUM(I,II)    sum of prob = 1
           CHOL(I,II)      Cholesky decomposition of Q
           QINVEQ(I,II)    Calculation Inverse of Q
          ;

*
* ---- entropy model equations -------------------------------------------
*

 OBJENT..           -(SUM((I,II,K),PELAS(I,II,K)*LOG(PELAS(I,II,K))))
                =E= ENTROPY;

 FOC(I)..           GM(I) - SUM(FJ,A(I,FJ) * LAMBDA(FJ))
                    - D(I) - SUM(II,Q(I,II) * X(II,'LAND'))
                =E= 0;

 RES(FJ)..          SUM(I, A(I,FJ) * X(I,'LAND'))
                =E= B(FJ);

 CHOL(I,II)..       SUM(III,L(I,III) * L(II,III))
                =E= Q(I,II);

 ELAS(I,II)..       SUM(K,PELAS(I,II,K)*SUPEL(I,II,K))
                =E= (QINV(I,II) - ((1/SUM((III,IIII),QINV(III,IIII))) *
                                   SUM(III,QINV(I,III))*SUM(III,QINV(III,II))))
                    *  (GM(II)/X(I,'LAND'));
```

```
 ELASUM(I,II)..    SUM(K,PELAS(I,II,K))
                =E= 1;

 QINVEQ(I,II)..    SUM(III,Q(I,III) * QINV(III,II))
                =E= IDENTITY(I,II);

*
* ---- set bounds and starting values ------------------------------------------
*

 D.LO(I)    = -10000;
 D.UP(I)    = 10000;

 Q.LO(I,II) = -10000;
 Q.LO(I,I)  = 0.001;
 Q.UP(I,II) = 10000;
 Q.L(I,II) = SVQ(I,II);

 QINV.L(I,II) = SVQI(I,II);

 L.LO(I,II) = -10000;
 L.UP(I,II) = 10000;
 L.FX(I,II)$(ord(I) lt ord(II)) = 0;
 L.L(I,II) = SVL(I,II);

 PELAS.LO(I,II,K) = 0.00001;
 PELAS.UP(I,II,K) = 1;
 PELAS.L(I,II,K)  = 0.5;

 ELAS.LO(I,II) = SUPEL(I,II,'1');
 ELAS.UP(I,II) = SUPEL(I,II,'2');
 ELAS.L(I,II)  = CENEL(I,II);

 LAMBDA.FX(FJ)  = SMIN(I,GM(I));

 MODEL ENTRO /OBJENT,FOC,RES,CHOL,ELAS,ELASUM,QINVEQ/;

 OPTION NLP=CONOPT2;
 ENTRO.Optfile = 2;
 SOLVE ENTRO USING NLP MAXIMIZING ENTROPY;


*-------------------------------------------------------------------
*
*       Define Calibrated model with non-linear objective function
*
*-------------------------------------------------------------------

 PARAMETERS  PARD(I)       Calibrated  D-parameters
             PARQ(I,II)    Calibrated  Q parameters
           ;

 PARD(I)     =  D.L(I);
 PARQ(I,II)  =  Q.L(I,II);

 VARIABLES  XCAL(I,FJ)     land allocation
            PROFIT         profit

 POSITIVE VARIABLES XCAL;

 EQUATIONS
            RESOURCEN(FJ)  Fixed resources
            PROFEQ         Profit definition
          ;

*
* ----  equations of calibrated model -----------------------------
*

 PROFEQ..           PROFIT
                =E= SUM((I),   GM(I) * XCAL(I,'LAND'))
                    - (SUM((I),PARD(I) * XCAL(I,'LAND')
                    + SUM(II,0.5 * XCAL(I,'LAND') * PARQ(I,II) * XCAL(II,'LAND'))));

 RESOURCEN(FJ)..    SUM(I, XCAL(I,FJ) )  =L= B(FJ);

 XCAL.LO(I,FJ) = 0.0;
 XCAL.UP(I,FJ) = 100*X(I,FJ);

 MODEL LEONCAL /PROFEQ,RESOURCEN/;

 OPTION NLP=CONOPT2;
 SOLVE LEONCAL USING NLP MAXIMIZING PROFIT;
 LEONCAL.Optfile = 2;

*-------------------------------------------------------------------
```

```
*
*       Check if model calibrates
*
*---------------------------------------------------------------

    PARAMETER    PERDIF(I,FJ)  Percent difference in input allocation;

    PERDIF(I,FJ) = (XCAL.L(I,FJ) - X(I,FJ)) * 100 / X(I,FJ) ;

    DISPLAY PERDIF ;

*---------------------------------------------------------------
*
*       Calculation of Elasticities
*
*---------------------------------------------------------------

  PARAMETER ELASCAL;

  LOOP(III,
*
* --- increase of gross margin by 1 percent
*
     GM(III) = GM(III) * 1.01;
     SOLVE LEONCAL USING NLP MAXIMIZING PROFIT;
*
* --- percentage change in land allocation
*
     ELASCAL(I,FJ,III) = (XCAL.L(I,FJ)  - X(I,FJ))/X(I,FJ) * 100.;
*
*
     GM(III) = GM(III) / 1.01;
  );

  DISPLAY ELASCAL;
```

CES Production Function Model – Monte Carlo Simulations (CESSIM.GMS)

```
$OFFLISTING
*-------------------------------------------------------------------------
*** CESSIM.GMS ***
*
* VERSION by Thomas Heckelei and Hendrik Wolff:
* - CES-PRODUCTION FUNCTIONS
* - "DATA" without any prior
* - "LAMT" with prior centered on the true LAMDA
* - "LAMF" with prior centered on PERCENT_LAM*LAMDA
*
*-------------------------------------------------------------------------

$OFFSYMLIST OFFSYMXREF
 OPTION LIMROW = 0
 OPTION LIMCOL = 0
 option iterlim =5000

*-------------------------------------------------------------------------

 SETS  I       Production processes         /COT Cotton,WHT Wheat,RI Rice/
       IS(I)   Subset of crops              /COT Cotton,WHT Wheat/
       J       Inputs                       /Land,Water,Capital,Chemical /
       FJ(J)   Fixed inputs                 /Land,Water/
       VJ(J)   Variable inputs              /Capital,Chemical/
       G       Observations                 /1*100/
       R(G)    Subset of Observation
       K       Support Points               /1,2/
       VERS    Versions of different estimation techniques /DATA,LAMT,LAMF/
       INFO    Information Type of the Results
               /TRUE_SIG, TRUE_ALP, TRUE_BET_L,TRUE_BET_W,TRUE_BET_CAP,TRUE_BET_CHEM, TRUE_NU,
                MEAN_SIG, MEAN_ALP, MEAN_BET_L,MEAN_BET_W,MEAN_BET_CAP,MEAN_BET_CHEM, MEAN_NU,
                BIAS_SIG, BIAS_ALP, BIAS_BET_L,BIAS_BET_W,BIAS_BET_CAP,BIAS_BET_CHEM, BIAS_NU,
                VAR_SIG ,  VAR_ALP,  VAR_BET_L, VAR_BET_W, VAR_BET_CAP, VAR_BET_CHEM, VAR_NU ,
                RMSE_SIG, RMSE_ALP, RMSE_BET_L,RMSE_BET_W,RMSE_BET_CAP,RMSE_BET_CHEM, RMSE_NU/
             ;
 ALIAS (I,I1,I2);
 ALIAS (R,R2);
 ALIAS (VJ,VJ2);
 ALIAS (FJ,FJ2);

 FILE CON /CON/;

*--- Simulation Steering for the Monte Carlo Experiments -----------------

 SCALAR NSMP       Number of envisaged Monte Carlo Repetitions
/500/;
 NSMP = 10;

 SET GG / 1*5/;
 PARAMETER NOBS(GG) / 1 5, 2 10, 3 20, 4 30, 5 50/;

* (Note: 1. = 100%)

 SCALAR PERCENT_INPUT_FJ  StdDev of fixed Input allocations                  /0.05/;
 SCALAR PERCENT_INPUT_VJ  Define StdDev of variable Input allocations        /0.05/;
 SCALAR PERCENT_S  Define StdDev of Supply                                   /0.1/;
 SCALAR SPR_LAM spread around Lambda                                         /0.2/;
 SCALAR PERCENT_LAM  Factor Lamopt is multiplied with for the "False" prior  /0.9/;
 SCALAR SIGMARULE std.dev by Sigmarule                                       /5/;
 SCALAR PERCENT_P  Define std.dev. of the output Prices                      /0.02/;

*-------------------------------------------------------------------------
*
*      DATA SET
*
*-------------------------------------------------------------------------

 PARAMETER P(I)   observed output prices
      /COT         2.924
       WHT         2.98
       RI          7.09/;

 PARAMETER W(VJ)    Unit prices of resouWes
       /CAPITAL    10.0
        CHEMICAL   10.0/;

 PARAMETER B(FJ)   Resource constraints
      /LAND        2.65
       WATER       8.69/;

 TABLE RC(I,J)     Unit prices of resources

                 LAND       WATER      CAPITAL   CHEMICAL
```

```
       COT       66.0       25.6         10.0      10.0
       WHT       33.0       25.6         10.0      10.0
       RI        49.0       25.6         10.0      10.0;

 TABLE X(I,J)      Observed resource use

                 LAND       WATER       CAPITAL   CHEMICAL
       COT       1.49       4.47         3.960     2.640
       WHT       0.62       1.14         1.980     1.320
       RI        0.54       3.08         2.940     1.960
      ;

*---------------------------------------------------------------
*
*       Define Model to generate data
*
*---------------------------------------------------------------

*  the true parameter

 PARAMETER  PALP(I)      Scale Parameters production function
      /COT      200.0
       WHT      100.0
       RI       50.0/;

 TABLE  PBET(I,J)    Input Share parameters
                 LAND       WATER      CAPITAL   CHEMICAL
       COT       0.6        0.1        0.2       0.1
       WHT       0.7        0.1        0.1       0.1
       RI        0.5        0.3        0.1       0.1;

 PARAMETER PSIG(I)      Substitution Elasticity
      /COT       0.8
       WHT       0.8
       RI        0.8/;

 PARAMETER PNU(I)       returns to scale
      /COT       0.6
       WHT       0.8
       RI        0.8/;

 PARAMETER PGAM(I)      '(SIGMA-1)/SIGMA to ease writing of equations';

       PGAM(I) = (PSIG(I)-1)/PSIG(I);

*---------------------------------------------------------------
*
*       Good Starting Values to Generate Data
*
*---------------------------------------------------------------
TABLE XOPTS(I,J) starting value expected input allocation

             Land       Water     Capital    Chemical

COT       1.275       2.386       7.799       4.480
WHT       0.243       0.402       0.756       0.756
RI        1.132       5.901       4.600       4.600;

PARAMETER SEXPS(I)  starting value expected supply

     /COT  309.626,   WHT 39.503, RI 96.670/;


*---------------------------------------------------------------
* Data Generation
*---------------------------------------------------------------

 PARAMETER PG(G,I)         Output Prices for the G Observations
           BS(FJ,G)        resource availabilities in simulation
          ;

 VARIABLES  GXN(G,I,J)     Resource allocation
            GPROFIT(G)     Total profit
            GSUPPLY(G,I)   Supply
            OBJ            Objective
           ;

 POSITIVE VARIABLES GXN,GSUPPLY;

 EQUATIONS
            GRES(G,FJ)      Fixed resources
            GPROF           Profit definition
            GPRDF(G,I)      Supply function
            OBJECTIVE       Objective function
          ;
*
* ---- declare equations -----------------------------------------
```

```
*

 GRES(R,FJ)..  SUM(I, GXN(R,I,FJ) ) =L= BS(FJ,R);

 GPRDF(R,I)..  GSUPPLY(R,I)  =E= PALP(I) *
                        ((SUM(J, PBET(I,J) * (GXN(R,I,J)**(PGAM(I)))))**(PNU(I)/PGAM(I)));

 GPROF..    OBJ     =E= SUM(R,

*                          *** revenue
                          SUM((I),    PG(R,I) * GSUPPLY(R,I))
*                          *** variable resource cost
                         -SUM((I,VJ), RC(I,VJ) * GXN(R,I,VJ) ))
                         ;

 MODEL CESGEN /GRES,GPRDF,GPROF/;
 CESGEN.Optfile = 3;
 CESGEN.workspace = 10;

 OPTION NLP=CONOPT3;
 OPTION SOLPRINT = OFF;

 PARAMETER XOPT(G,I,J)     optimal input allocation
           SEXP(G,I)       expected supply
           LAMOPT(G,FJ)    true shadow prices
          ;

*---------------------------------------------------------------------
*
*       Define Model for Entropy-Estimation
*
*---------------------------------------------------------------------

 PARAMETER
      SOBS(G,I)         Observed supply
      XOBS(G,I,J)       Observed input allocations
      TXQ(G,VJ)         Total variable input quantities
      SPRERRS(I)        spread supply support errors
      SPRERRFJ(I,FJ)    spread input supports errors
      SPRERRVJ(I,VJ)    spread input supports errors
      SPRX(G,I,VJ)      spread input supports errors
      SUPERRS(I,K)      supports supply errors
      SUPERRFJ(I,FJ,K)  supports input errors
      SUPERRVJ(I,VJ,K)  supports input errors
      SUPLAMT(G,FJ,K)   supports shadow prices
      SUPLAMF(G,FJ,K)   supports shadow prices
      SUPX(G,I,VJ,K)    supports variable inputs
      CENERR            center error supports /0.0/
      CENLAMT(G,FJ)     center lambda supports
      CENLAMF(G,FJ)     center lambda supports
          ;

 VARIABLES  ENTROPY
           ALPHA(I)            Scale Parameters production function
           BETA(I,J)           Input Share parameters
           SIGMA(I)            Substitution Elasticity
           GAMMA(I)            '(SIGMA-1)/SIGMA to ease writing of equations'
           NU(I)               returns to scale
           PERRS(G,I,K)        Probabilities for supply Error-supports
           PERRFJ(G,I,FJ,K)    Probabilities for fixed input Error-supports
           PERRVJ(G,I,VJ,K)    Probabilities for variable input Error-supports
           PLAM(G,FJ,K)        Probabilities for Shadow price supports
           PX(G,I,VJ,K)        Probabilities for variable input supports
           LAMBDA(G,FJ)        Shadow prices of fixed resources
           ERRORS(G,I)         Disturbance terms supply
           ERRFJ(G,I,FJ)       Disturbance terms fixed input allocation
           ERRVJ(G,I,VJ)       Disturbance terms variable input allocation
          ;

 POSITIVE VARIABLES PERRS,PERRFJ,PERRVJ,PLAM,PX,LAMBDA
          ;

 EQUATIONS
           OBJENT_LAM        Objective Function
           OBJENT_DATA       Objective Function
           PRODFC(G,I)       CES-Production Functions
           MVP(G,I,J)        FOC-variable inputs
           FJALLO(G,I,FJ)    FOC-fixed Input allocation equations
           GAMSUB(I)         Relationship Gamma to Sigma
           SUMBET(I)         Betas sum to one
           EXPERR(G,I)       Error =  sum over prob * supports
           EXPERRFJ(G,I,FJ)  Error =  sum over prob * supports
           EXPERRVJ(G,I,VJ)  Error =  sum over prob * supports
           EXPLAMT(G,FJ)     Lambda =  sum over prob * supports
           EXPLAMF(G,FJ)     Lambda =  sum over prob * supports
           ERRSSUM(G,I)      sum of prob = 1
           ERRFJSUM(G,I,FJ)  sum of prob = 1
```

```
           ERRVJSUM(G,I,VJ) sum of prob = 1
           LAMSUM(G,FJ)     sum of prob = 1
           RES(G,FJ)        sum of input allocations = total quantity
          ;
*
* ---- declare equations ----------------------------------------
*

 OBJENT_LAM..           - (SUM((R,I,K),PERRS(R,I,K)      * LOG(PERRS(R,I,K)))
                        + SUM((R,I,VJ,K),PERRVJ(R,I,VJ,K) * LOG(PERRVJ(R,I,VJ,K)))
                        + SUM((R,I,FJ,K),PERRFJ(R,I,FJ,K) * LOG(PERRFJ(R,I,FJ,K)))
                        + (SUM((R,FJ,K),PLAM(R,FJ,K)      * LOG(PLAM(R,FJ,K)))))
                =E= ENTROPY;

 OBJENT_DATA..           - (SUM((R,I,K),PERRS(R,I,K) * LOG(PERRS(R,I,K)))
                        + SUM((R,I,FJ,K),PERRFJ(R,I,FJ,K) * LOG(PERRFJ(R,I,FJ,K)))
                        + SUM((R,I,VJ,K),PERRVJ(R,I,VJ,K) * LOG(PERRVJ(R,I,VJ,K))))
                =E= ENTROPY;

 PRODFC(R,I)..      ALPHA(I) *
                    ((SUM(VJ, BETA(I,VJ) * ((XOBS(R,I,VJ) - ERRVJ(R,I,VJ))**(GAMMA(I))))
                    + SUM(FJ, BETA(I,FJ) * ((XOBS(R,I,FJ) - ERRFJ(R,I,FJ))**(GAMMA(I))))
                    )**(NU(I)/GAMMA(I)))
                =E= SOBS(R,I) - ERRORS(R,I);

 MVP(R,I,VJ)..      PG(R,I) * ALPHA(I) * (NU(I)/GAMMA(I))
                    * ((SUM(VJ2, BETA(I,VJ2) * ((XOBS(R,I,VJ2) - ERRVJ(R,I,VJ2))**(GAMMA(I))))
                    + SUM(FJ, BETA(I,FJ) * ((XOBS(R,I,FJ) - ERRFJ(R,I,FJ))**(GAMMA(I))))
                    )**((NU(I)/GAMMA(I))-1))
                    * GAMMA(I)*BETA(I,VJ)*((XOBS(R,I,VJ) - ERRVJ(R,I,VJ))**(GAMMA(I)-1))
                =E= RC(I,VJ);

 FJALLO(R,I,FJ)..   PG(R,I) * ALPHA(I) * (NU(I)/GAMMA(I))
                    * ((SUM(VJ, BETA(I,VJ) * ((XOBS(R,I,VJ) - ERRVJ(R,I,VJ))**(GAMMA(I))))
                    + SUM(FJ2, BETA(I,FJ2) * ((XOBS(R,I,FJ2) - ERRFJ(R,I,FJ2))**(GAMMA(I))))
                    )**((NU(I)/GAMMA(I))-1))
                    * GAMMA(I)*BETA(I,FJ)*((XOBS(R,I,FJ) - ERRFJ(R,I,FJ))**(GAMMA(I)-1))
                =E= LAMBDA(R,FJ);

 GAMSUB(I)..        GAMMA(I) =E= (SIGMA(I)-1)/SIGMA(I);

 SUMBET(I)..        SUM(J,BETA(I,J)) =E= 1;

 EXPERR(R,I)..      ERRORS(R,I)   =E= SUM(K,PERRS(R,I,K)*SUPERRS(I,K));

 EXPERRFJ(R,I,FJ).. ERRFJ(R,I,FJ) =E= SUM(K,PERRFJ(R,I,FJ,K)*SUPERRFJ(I,FJ,K));

 EXPERRVJ(R,I,VJ).. ERRVJ(R,I,VJ) =E= SUM(K,PERRVJ(R,I,VJ,K)*SUPERRVJ(I,VJ,K));

 EXPLAMT(R,FJ)..    LAMBDA(R,FJ) =E= SUM(K,PLAM(R,FJ,K)*SUPLAMT(R,FJ,K));

 EXPLAMF(R,FJ)..    LAMBDA(R,FJ) =E= SUM(K,PLAM(R,FJ,K)*SUPLAMF(R,FJ,K));

 ERRSSUM(R,I)..     SUM(K,PERRS(R,I,K))     =E= 1;

 ERRFJSUM(R,I,FJ).. SUM(K,PERRFJ(R,I,FJ,K)) =E= 1;

 ERRVJSUM(R,I,VJ).. SUM(K,PERRVJ(R,I,VJ,K)) =E= 1;

 LAMSUM(R,FJ)..     SUM(K,PLAM(R,FJ,K)) =E= 1;

 RES(R,FJ)..        SUM(I, XOBS(R,I,FJ) - ERRFJ(R,I,FJ)) =E= BS(FJ,R);


 MODEL ME_LAMT  /OBJENT_LAM, PRODFC,  MVP,  FJALLO,  GAMSUB,SUMBET,EXPERR,EXPERRFJ,
                 EXPERRVJ,EXPLAMT,ERRSSUM,ERRFJSUM,ERRVJSUM,LAMSUM,RES/;
 MODEL ME_LAMF  /OBJENT_LAM, PRODFC,  MVP,  FJALLO,  GAMSUB,SUMBET,EXPERR,EXPERRFJ,
                 EXPERRVJ,EXPLAMF,ERRSSUM,ERRFJSUM,ERRVJSUM,LAMSUM,RES/;
 MODEL ME_DATA  /OBJENT_DATA,PRODFC,  MVP,  FJALLO,  GAMSUB,SUMBET,EXPERR,EXPERRFJ,
                 EXPERRVJ, ERRSSUM,ERRFJSUM,ERRVJSUM,RES/;

*****************************************************************************************
*          Monte Carlo
*
*****************************************************************************************
 PARAMETER CCOUNT(GG)   Counts the number of MC repetition up to NSMP
 PARAMETER NINFES(GG,VERS)   Counts the number of not optimal solutions;
 PARAMETER CHECK /0/;
*
* ---- monte carlo loop setup  ---------------------------------------------
*

 PARAMETER
           SMPERRS(G,I)               Sampling Errors supply
           SMPERRFJ(G,I,FJ)           Sampling Errors inputs
           SMPERRVJ(G,I,VJ)           Sampling Errors inputs
           MSOBS(I)                   Mean observed supply
```

```
           MXOBS(I,FJ)               Mean observed inputs
           STDDERRS(I)               Standard deviation supply errors
           STDDERRFJ(I,FJ)           Standard deviation input errors
           STDDERRVJ(I,VJ)           Standard deviation input errors
           SIGSAVE(VERS,I)           Intermediate save sigma
           ALPSAVE(VERS,I)           Intermediate save alpha
           BETSAVE(VERS,I,J )        Intermediate save beta
           NUSAVE(VERS,I)            Intermediate save nu
           LAMSAVE(VERS,G,FJ)        Intermediate save lambda
           MEANSIG(VERS,GG,I)        Mean of Sigma estimates
           MEANALP(VERS,GG,I)        Mean of Alpha estimates
           MEANBET(VERS,GG,I,J)      Mean of Beta estimates
           MEANNU(VERS,GG,I)         Mean of Beta estimates
           BIASSIG(VERS,GG,I)        Bias of Sigma estimates
           BIASALP(VERS,GG,I)        Bias of Alpha estimates
           BIASBET(VERS,GG,I,J)      Bias of Beta estimates
           BIASNU(VERS,GG,I)         Bias of Beta estimates
           BIASLAM(VERS,GG,G,FJ)     Bias of Lambda estimates
           VARSIG(VERS,GG,I)         Variance of Sigma estimates
           VARALP(VERS,GG,I)         Variance of Alpha estimates
           VARBET(VERS,GG,I,J)       Variance of Beta estimates
           VARNU(VERS,GG,I)          Variance of Beta estimates
           MSELAM(VERS,GG,G,FJ)      MSE of Lambda estimates
           RMSESIG(VERS,GG,I)        RMSE of Sigma estimates
           RMSEALP(VERS,GG,I)        RMSE of Alpha estimates
           RMSEBET(VERS,GG,I,J)      RMSE of Beta estimates
           RMSENU(VERS,GG,I)         RMSE of Beta estimates
           RMSELAM(VERS,GG,G,FJ)     RMSE of Lambda estimates
           RMSEX(VERS,GG,G,I,VJ)     RMSE of estimated input quantities
          ;

*
*  LOOP over the experiments ------------------------------------------------------------------
*
 BS('WATER',G) = B('WATER');
 SIGSAVE(VERS,I)     =  0;
 ALPSAVE(VERS,I)     =  0;
 BETSAVE(VERS,I,J )  =  0;
 NUSAVE(VERS,I)      =  0;
 LAMSAVE(VERS,G,FJ)  =  0;

 MEANSIG(VERS,GG,I)     = 0;
 MEANALP(VERS,GG,I)     = 0;
 MEANBET(VERS,GG,I,J)   = 0;
 MEANNU(VERS,GG,I)      = 0;
 VARSIG(VERS,GG,I)      = 0;
 VARALP(VERS,GG,I)      = 0;
 VARBET(VERS,GG,I,J)    = 0;
 VARNU(VERS,GG,I)       = 0;

 BIASLAM(VERS,GG,G,FJ) = 0;
 MSELAM(VERS,GG,G,FJ)  = 0;

 CCOUNT(GG)    = 0;
 NINFES(GG,VERS)     = 0;

 LOOP(GG,
     R(G)  = No;
     R(G) $ (ORD(G) LE NOBS(GG)) = YES;

     WHILE(CCOUNT(GG) LT NSMP,

*-------------------------------------------------------------------
*
*       Generate true data
*
*-------------------------------------------------------------------

         PG(R,I) = P(I) +  NORMAL(0,PERCENT_P*P(I));

         BS(FJ,R) = B(FJ) + NORMAL(0,0.1);

         GXN.LO(R,I,J)    = 0.00001;
         GXN.UP(R,I,FJ)   = BS(FJ,R);
         GXN.UP(R,I,VJ)   = 20;
         GXN.L(R,I,J)     = XOPTS(I,J);

         GSUPPLY.LO(R,I) = 0.00001;
         GSUPPLY.UP(R,I) = 500;
         GSUPPLY.L(R,I)  = SEXPS(I);

         putclose CON /// "Solving for CESGEN for size ",GG.tl ////;

         SOLVE CESGEN USING NLP MAXIMIZING OBJ;

         XOPT(R,I,J)       = GXN.L(R,I,J);
         SEXP(R,I)         = GSUPPLY.L(R,I);
```

```
         LAMOPT(R,FJ)     = GRES.M(R,FJ);

         TXQ(R,VJ)               = SUM(I,XOPT(R,I,VJ));
         CENLAMT(R,FJ)           = LAMOPT(R,FJ);
         SUPLAMT(R,FJ,'1')    = (1 - SPR_LAM) * CENLAMT(R,FJ);
         SUPLAMT(R,FJ,'2')    = (1 + SPR_LAM) * CENLAMT(R,FJ);

         CENLAMF(R,FJ)           = PERCENT_LAM*LAMOPT(R,FJ);
         SUPLAMF(R,FJ,'1')    = (1 - SPR_LAM) * CENLAMF(R,FJ);
         SUPLAMF(R,FJ,'2')    = (1 + SPR_LAM) * CENLAMF(R,FJ);

*-------------------------------------------------------------
*
*       Introduce Errors
*
*-------------------------------------------------------------
         STDDERRS(I)          = (SUM(R,SEXP(R,I))/CARD(R))     * PERCENT_S;
         STDDERRFJ(I,FJ)      = (SUM(R,XOPT(R,I,FJ))/CARD(R))  * PERCENT_INPUT_FJ;
         STDDERRVJ(I,VJ)      = (SUM(R,XOPT(R,I,VJ))/CARD(R))  * PERCENT_INPUT_VJ;

         SMPERRS(R,I)           = NORMAL(0,STDDERRS(I));

         SMPERRFJ(R,IS,FJ)      = NORMAL(0,STDDERRFJ(IS,FJ));
         SMPERRFJ(R,'RI',FJ)    = -SUM(IS,SMPERRFJ(R,IS,FJ));

         SMPERRVJ(R,IS,VJ)      = NORMAL(0,STDDERRVJ(IS,VJ));
         SMPERRVJ(R,'RI',VJ)    = -SUM(IS,SMPERRVJ(R,IS,VJ));

         SOBS(R,I)              = SEXP(R,I)     + SMPERRS(R,I);
         XOBS(R,I,FJ)           = XOPT(R,I,FJ)  + SMPERRFJ(R,I,FJ);
         XOBS(R,I,VJ)           = XOPT(R,I,VJ)  + SMPERRVJ(R,I,VJ);
         MSOBS(I)               = SUM(R,SOBS(R,I))/CARD(R);
         MXOBS(I,FJ)            = SUM(R,XOBS(R,I,FJ))/CARD(R);

         SPRERRS(I)             = SIGMARULE*STDDERRS(I);
         SPRERRFJ(I,FJ)         = SIGMARULE*STDDERRFJ(I,FJ);
         SPRERRVJ(I,VJ)         = SIGMARULE*STDDERRVJ(I,VJ);

         SUPERRS(I,'1')         = CENERR - SPRERRS(I);
         SUPERRS(I,'2')         = CENERR + SPRERRS(I);
         SUPERRFJ(I,FJ,'1')     = CENERR - SPRERRFJ(I,FJ);
         SUPERRFJ(I,FJ,'2')     = CENERR + SPRERRFJ(I,FJ);
         SUPERRVJ(I,VJ,'1')     = CENERR - SPRERRVJ(I,VJ);
         SUPERRVJ(I,VJ,'2')     = CENERR + SPRERRVJ(I,VJ);

         LOOP (VERS,
            putclose CON //// "Solving for ",VERS.tl," and size ",GG.tl, " in CCOUNT Loop " ,
                             CCOUNT(GG) ////;
*
* ---- set bounds and starting values -----------------------------------------
*

              PERRS.LO(G,I,K) = 0.00001;
              PERRS.UP(G,I,K) = 1;

              PERRFJ.LO(G,I,FJ,K) = 0.00001;
              PERRFJ.UP(G,I,FJ,K) = 1;

              PERRVJ.LO(G,I,VJ,K) = 0.00001;
              PERRVJ.UP(G,I,VJ,K) = 1;

              PLAM.LO(G,FJ,K) = 0.0000001;
              PLAM.UP(G,FJ,K) = 1;

              ALPHA.L(I)   = PALP(I);
              ALPHA.LO(I)  = 0.000001;
              ALPHA.UP(I)  = 10000;

              BETA.L(I,J)  = PBET(I,J);
              BETA.LO(I,J) = 0.0000001;
              BETA.UP(I,J) = 1;

              SIGMA.L(I)   = PSIG(I);
              SIGMA.LO(I)  = 0.00001;
              SIGMA.UP(I)  = 30;

              NU.L(I)      = PNU(I);
              NU.LO(I)     = 0.1;
              NU.UP(I)     = 1;

              GAMMA.L(I)   = PGAM(I);
              GAMMA.UP(I)  = -0.01;
              GAMMA.LO(I)  = -1;

              IF (SAMEAS(VERS,"LAMT"),
                PLAM.L(R,FJ,'1') = (LAMOPT(R,FJ)-SUPLAMT(R,FJ,'2'))/(SUPLAMT(R,FJ,'1')-
                                   SUPLAMT(R,FJ,'2'));
```

```
  PLAM.L(R,FJ,'2') = 1-PLAM.L(R,FJ,'1');
  LAMBDA.L(R,FJ)   = LAMOPT(R,FJ);
  LAMBDA.LO(R,FJ)  = 0.000001;
  LAMBDA.UP(R,FJ)  = SUPLAMT(R,FJ,'2');
 );

IF (SAMEAS(VERS,"LAMF"),
  PLAM.L(R,FJ,'1') = (LAMOPT(R,FJ)-SUPLAMF(R,FJ,'2'))/(SUPLAMF(R,FJ,'1')-
                      SUPLAMF(R,FJ,'2'));
  PLAM.L(R,FJ,'2') = 1-PLAM.L(R,FJ,'1');
  LAMBDA.L(R,FJ)   = LAMOPT(R,FJ);
  LAMBDA.LO(R,FJ)  = 0.000001;
  LAMBDA.UP(R,FJ)  = SUPLAMF(R,FJ,'2');
 );

IF (SAMEAS(VERS,"DATA"),
  LAMBDA.L(R,FJ)   = LAMOPT(R,FJ);
  LAMBDA.LO(R,FJ)  = 0.000001;
  LAMBDA.UP(R,FJ)  = SUPLAMT(R,FJ,'2');
 );

ME_LAMF.Optfile = 3;
ME_LAMT.Optfile = 3;
ME_DATA.Optfile = 3;

OPTION NLP=CONOPT3;

PERRS.L(R,I,'1')  = (SMPERRS(R,I)-SUPERRS(I,'2'))/(SUPERRS(I,'1')-
                    SUPERRS(I,'2'));
PERRS.L(R,I,'2')  = 1-PERRS.L(R,I,'1');
ERRORS.L(R,I)     = SUM(K,PERRS.L(R,I,K) * SUPERRS(I,K));
ERRORS.LO(R,I)    = SUPERRS(I,'1');
ERRORS.UP(R,I)    = SUPERRS(I,'2');

PERRFJ.L(R,I,FJ,'1') = (SMPERRFJ(R,I,FJ)-SUPERRFJ(I,FJ,'2'))/(SUPERRFJ(I,FJ,'1')-
                        SUPERRFJ(I,FJ,'2'));
PERRFJ.L(R,I,FJ,'2') = 1-PERRFJ.L(R,I,FJ,'1');
ERRFJ.L(R,I,FJ)       = SUM(K,PERRFJ.L(R,I,FJ,K) * SUPERRFJ(I,FJ,K));
ERRFJ.LO(R,I,FJ)      = SUPERRFJ(I,FJ,'1');
ERRFJ.UP(R,I,FJ)      = SUPERRFJ(I,FJ,'2');

PERRVJ.L(R,I,VJ,'1') = (SMPERRVJ(R,I,VJ)-SUPERRVJ(I,VJ,'2'))/(SUPERRVJ(I,VJ,'1')-
                        SUPERRVJ(I,VJ,'2'));
PERRVJ.L(R,I,VJ,'2') = 1-PERRVJ.L(R,I,VJ,'1');
ERRVJ.L(R,I,VJ)       = SUM(K,PERRVJ.L(R,I,VJ,K) * SUPERRVJ(I,VJ,K));
ERRVJ.L(R,I,VJ)       = SUM(K,PERRVJ.L(R,I,VJ,K) * SUPERRVJ(I,VJ,K));
ERRVJ.LO(R,I,VJ)      = SUPERRVJ(I,VJ,'1');
ERRVJ.UP(R,I,VJ)      = SUPERRVJ(I,VJ,'2');

IF (SAMEAS(VERS,"LAMT"),
    SOLVE ME_LAMT USING NLP MAXIMIZING ENTROPY;
    IF ((ME_LAMT.modelstat GE 3),
        NINFES(GG,VERS) = NINFES(GG,VERS) + 1;
        CHECK = 1;
    ELSE
        SIGSAVE(VERS,I)    =  SIGMA.L(I);
        ALPSAVE(VERS,I)    =  ALPHA.L(I);
        BETSAVE(VERS,I,J ) =  BETA.L(I,J);
        NUSAVE(VERS,I)     =  NU.L(I);
        LAMSAVE(VERS,R,FJ) =  LAMBDA.L(R,FJ);
    );
);

IF (SAMEAS(VERS,"LAMF") AND (CHECK EQ 0),
    SOLVE ME_LAMF USING NLP MAXIMIZING ENTROPY;
    IF ((ME_LAMF.modelstat GE 3),
        NINFES(GG,VERS) = NINFES(GG,VERS) + 1;
        CHECK = 1;
    ELSE
        SIGSAVE(VERS,I)    =  SIGMA.L(I);
        ALPSAVE(VERS,I)    =  ALPHA.L(I);
        BETSAVE(VERS,I,J ) =  BETA.L(I,J);
        NUSAVE(VERS,I)     =  NU.L(I);
        LAMSAVE(VERS,R,FJ) =  LAMBDA.L(R,FJ);
    );
);

IF (SAMEAS(VERS,"DATA") AND (CHECK EQ 0),
    SOLVE ME_DATA USING NLP MAXIMIZING ENTROPY;
    IF ((ME_DATA.modelstat GE 3),
        NINFES(GG,VERS) = NINFES(GG,VERS) + 1;
        CHECK = 1;
    ELSE
        SIGSAVE(VERS,I)    =  SIGMA.L(I);
        ALPSAVE(VERS,I)    =  ALPHA.L(I);
        BETSAVE(VERS,I,J ) =  BETA.L(I,J);
        NUSAVE(VERS,I)     =  NU.L(I);
```

```
                    LAMSAVE(VERS,R,FJ) =  LAMBDA.L(R,FJ);
                );
            );
        );

DISPLAY CHECK;
        IF (CHECK EQ 0,
            MEANSIG(VERS,GG,I)    = MEANSIG(VERS,GG,I)       + SIGSAVE(VERS,I);
            MEANALP(VERS,GG,I)    = MEANALP(VERS,GG,I)       + ALPSAVE(VERS,I);
            MEANBET(VERS,GG,I,J) = MEANBET(VERS,GG,I,J)     + BETSAVE(VERS,I,J);
            MEANNU(VERS,GG,I)     = MEANNU(VERS,GG,I)        + NUSAVE(VERS,I);
            BIASLAM(VERS,GG,R,FJ)= BIASLAM(VERS,GG,R,FJ)    + (LAMSAVE(VERS,R,FJ) -
LAMOPT(R,FJ));
            VARSIG(VERS,GG,I)     = VARSIG(VERS,GG,I)        + SIGSAVE(VERS,I)*SIGSAVE(VERS,I);
            VARALP(VERS,GG,I)     = VARALP(VERS,GG,I)        + ALPSAVE(VERS,I)*ALPSAVE(VERS,I);
            VARBET(VERS,GG,I,J)  = VARBET(VERS,GG,I,J)      +
BETSAVE(VERS,I,J)*BETSAVE(VERS,I,J);
            VARNU(VERS,GG,I)      = VARNU(VERS,GG,I)         + NUSAVE(VERS,I)*NUSAVE(VERS,I);
            MSELAM(VERS,GG,R,FJ) = MSELAM(VERS,GG,R,FJ) + SQR(LAMSAVE(VERS,R,FJ) -
LAMOPT(R,FJ));
            CCOUNT(GG)= CCOUNT(GG) + 1;
        );

DISPLAY CCOUNT;
        CHECK = 0;
    );
 );


* Calculation of Means, Variances, Bias and RMSE

MEANSIG(VERS,GG,I)    = MEANSIG(VERS,GG,I)/CCOUNT(GG);
MEANALP(VERS,GG,I)    = MEANALP(VERS,GG,I)/CCOUNT(GG);
MEANBET(VERS,GG,I,J) = MEANBET(VERS,GG,I,J)/CCOUNT(GG);
MEANNU(VERS,GG,I)     = MEANNU(VERS,GG,I)/CCOUNT(GG);

BIASSIG(VERS,GG,I)    = MEANSIG(VERS,GG,I)       -PSIG(I);
BIASALP(VERS,GG,I)    = MEANALP(VERS,GG,I)       -PALP(I);
BIASBET(VERS,GG,I,J) = MEANBET(VERS,GG,I,J)     -PBET(I,J);
BIASNU(VERS,GG,I)     = MEANNU(VERS,GG,I)        -PNU(I);
BIASLAM(VERS,GG,G,FJ) $(BIASLAM(VERS,GG,G,FJ)) = BIASLAM(VERS,GG,G,FJ)/CCOUNT(GG);

VARSIG(VERS,GG,I)     = SQRT(SQR(VARSIG(VERS,GG,I)/CCOUNT(GG)
                        - MEANSIG(VERS,GG,I) * MEANSIG(VERS,GG,I)));
VARALP(VERS,GG,I)     = SQRT(SQR(VARALP(VERS,GG,I)/CCOUNT(GG)
                        - MEANALP(VERS,GG,I) * MEANALP(VERS,GG,I)));
VARBET(VERS,GG,I,J)  = SQRT(SQR(VARBET(VERS,GG,I,J)/CCOUNT(GG)
                        - MEANBET(VERS,GG,I,J) * MEANBET(VERS,GG,I,J)));
VARNU(VERS,GG,I)      = SQRT(SQR(VARNU(VERS,GG,I)/CCOUNT(GG)
                        - MEANNU(VERS,GG,I) * MEANNU(VERS,GG,I)));

RMSESIG(VERS,GG,I)    = SQRT(SQRT(SQR(VARSIG(VERS,GG,I) + BIASSIG(VERS,GG,I) *
BIASSIG(VERS,GG,I))));
RMSEALP(VERS,GG,I)    = SQRT(SQRT(SQR(VARALP(VERS,GG,I) + BIASALP(VERS,GG,I) *
BIASALP(VERS,GG,I))));
RMSEBET(VERS,GG,I,J) = SQRT(SQRT(SQR(VARBET(VERS,GG,I,J)
                        + BIASBET(VERS,GG,I,J) * BIASBET(VERS,GG,I,J))));
RMSENU(VERS,GG,I)     = SQRT(SQRT(SQR(VARNU(VERS,GG,I) + BIASNU(VERS,GG,I) * BIASNU(VERS,GG,I))));
RMSELAM(VERS,GG,G,FJ) $(MSELAM(VERS,GG,G,FJ))= SQRT(MSELAM(VERS,GG,G,FJ)/CCOUNT(GG));

DISPLAY MEANSIG,BIASSIG,VARSIG;
DISPLAY MEANALP,BIASALP,VARALP;
DISPLAY MEANBET,BIASBET,VARBET;
DISPLAY MEANNU , BIASNU, VARNU;
DISPLAY BIASLAM,RMSELAM;
DISPLAY RMSESIG, RMSEALP, RMSEBET, RMSENU;

PARAMETER SNRMSE(VERS,GG)     Sum of normalized RMSE;
PARAMETER SRMSE(VERS,GG)      Sum of RMSE;
PARAMETER SABIAS(VERS,GG)     Sum of the absolute BIAS;
PARAMETER SSTD(VERS,GG)       Sum of the Standard Deviation;
PARAMETER AVBIASL(VERS,GG,FJ)   Average bias of Lambda;
PARAMETER AVRMSEL(VERS,GG,FJ)   Average RMSE of Lambda;

SNRMSE(VERS,GG) =   SUM(I,RMSESIG(VERS,GG,I)/ABS(PSIG(I)))
                  + SUM(I,RMSEALP(VERS,GG,I)/ABS(PALP(I)))
                  + SUM((I,J),RMSEBET(VERS,GG,I,J)/ABS(PBET(I,J)))
                  + SUM(I,RMSENU(VERS,GG,I)/ABS(PNU(I)));

SRMSE(VERS,GG) =    SUM(I,RMSESIG(VERS,GG,I))
                  + SUM(I,RMSEALP(VERS,GG,I))
                  + SUM((I,J),RMSEBET(VERS,GG,I,J))
                  + SUM(I,RMSENU(VERS,GG,I));

SABIAS(VERS,GG) =   SUM(I,ABS(BIASSIG(VERS,GG,I)))
                  + SUM(I,ABS(BIASALP(VERS,GG,I)))
                  + SUM((I,J),ABS(BIASBET(VERS,GG,I,J)))
```

```
                  + SUM(I,ABS(BIASNU(VERS,GG,I)));

SSTD(VERS,GG)=      SUM(I,SQRT(VARSIG(VERS,GG,I)))
                  + SUM(I,SQRT(VARALP(VERS,GG,I)))
                  + SUM((I,J),SQRT(VARBET(VERS,GG,I,J)))
                  + SUM(I,SQRT(VARNU(VERS,GG,I)));

AVBIASL(VERS,GG,FJ) = SUM(G,BIASLAM(VERS,GG,G,FJ))/SUM(G $ BIASLAM(VERS,GG,G,FJ),1.);
AVRMSEL(VERS,GG,FJ) = SUM(G,RMSELAM(VERS,GG,G,FJ))/SUM(G $ RMSELAM(VERS,GG,G,FJ),1.);

DISPLAY SNRMSE,SRMSE,SABIAS,SSTD,AVBIASL,AVRMSEL;

PARAMETER RESULT(*,VERS,GG,I);

RESULT("TRUE_SIG",VERS,GG,I)          = PSIG(I)   ;
RESULT("TRUE_ALP",VERS,GG,I)          = PALP(I)   ;
RESULT("TRUE_BET_L"   ,VERS,GG,I)     = PBET(I,"LAND") ;
RESULT("TRUE_BET_W"   ,VERS,GG,I)     = PBET(I,"WATER") ;
RESULT("TRUE_BET_CAP" ,VERS,GG,I)     = PBET(I,"CAPITAL") ;
RESULT("TRUE_BET_CHEM",VERS,GG,I)     = PBET(I,"CHEMICAL") ;
RESULT("TRUE_NU ",VERS,GG,I)          = PNU(I)  ;

RESULT("MEAN_SIG",VERS,GG,I)          = MEANSIG(VERS,GG,I)   ;
RESULT("MEAN_ALP",VERS,GG,I)          = MEANALP(VERS,GG,I)   ;
RESULT("MEAN_BET_L"   ,VERS,GG,I)     = MEANBET(VERS,GG,I,"LAND")     ;
RESULT("MEAN_BET_W"   ,VERS,GG,I)     = MEANBET(VERS,GG,I,"WATER")    ;
RESULT("MEAN_BET_CAP" ,VERS,GG,I)     = MEANBET(VERS,GG,I,"CAPITAL")  ;
RESULT("MEAN_BET_CHEM",VERS,GG,I)     = MEANBET(VERS,GG,I,"CHEMICAL") ;
RESULT("MEAN_NU ",VERS,GG,I)          = MEANNU(VERS,GG,I)    ;

RESULT("BIAS_SIG",VERS,GG,I)          = BIASSIG(VERS,GG,I)   ;
RESULT("BIAS_ALP",VERS,GG,I)          = BIASALP(VERS,GG,I)   ;
RESULT("BIAS_BET_L"   ,VERS,GG,I)     = BIASBET(VERS,GG,I,"LAND")     ;
RESULT("BIAS_BET_W"   ,VERS,GG,I)     = BIASBET(VERS,GG,I,"WATER")    ;
RESULT("BIAS_BET_CAP" ,VERS,GG,I)     = BIASBET(VERS,GG,I,"CAPITAL")  ;
RESULT("BIAS_BET_CHEM",VERS,GG,I)     = BIASBET(VERS,GG,I,"CHEMICAL") ;
RESULT("BIAS_NU ",VERS,GG,I)          = BIASNU(VERS,GG,I)    ;

RESULT("ABSBIAS_SIG",VERS,GG,I)          = ABS(BIASSIG(VERS,GG,I)   );
RESULT("ABSBIAS_ALP",VERS,GG,I)          = ABS(BIASALP(VERS,GG,I)   );
RESULT("ABSBIAS_BET_L"   ,VERS,GG,I)     = ABS(BIASBET(VERS,GG,I,"LAND")    ) ;
RESULT("ABSBIAS_BET_W"   ,VERS,GG,I)     = ABS(BIASBET(VERS,GG,I,"WATER")   ) ;
RESULT("ABSBIAS_BET_CAP" ,VERS,GG,I)     = ABS(BIASBET(VERS,GG,I,"CAPITAL") ) ;
RESULT("ABSBIAS_BET_CHEM",VERS,GG,I)     = ABS(BIASBET(VERS,GG,I,"CHEMICAL")) ;
RESULT("ABSBIAS_NU ",VERS,GG,I)          = ABS(BIASNU(VERS,GG,I)    );

RESULT("VAR_SIG",VERS,GG,I)          = VARSIG(VERS,GG,I)   ;
RESULT("VAR_ALP",VERS,GG,I)          = VARALP(VERS,GG,I)   ;
RESULT("VAR_BET_L"   ,VERS,GG,I)     = VARBET(VERS,GG,I,"LAND")     ;
RESULT("VAR_BET_W"   ,VERS,GG,I)     = VARBET(VERS,GG,I,"WATER")    ;
RESULT("VAR_BET_CAP" ,VERS,GG,I)     = VARBET(VERS,GG,I,"CAPITAL")  ;
RESULT("VAR_BET_CHEM",VERS,GG,I)     = VARBET(VERS,GG,I,"CHEMICAL") ;
RESULT("VAR_NU ",VERS,GG,I)          = VARNU(VERS,GG,I)    ;

RESULT("RMSE_SIG",VERS,GG,I)          = RMSESIG(VERS,GG,I)   ;
RESULT("RMSE_ALP",VERS,GG,I)          = RMSEALP(VERS,GG,I)   ;
RESULT("RMSE_BET_L"   ,VERS,GG,I)     = RMSEBET(VERS,GG,I,"LAND")     ;
RESULT("RMSE_BET_W"   ,VERS,GG,I)     = RMSEBET(VERS,GG,I,"WATER")    ;
RESULT("RMSE_BET_CAP" ,VERS,GG,I)     = RMSEBET(VERS,GG,I,"CAPITAL")  ;
RESULT("RMSE_BET_CHEM",VERS,GG,I)     = RMSEBET(VERS,GG,I,"CHEMICAL") ;
RESULT("RMSE_NU ",VERS,GG,I)          = RMSENU(VERS,GG,I)    ;

DISPLAY RESULT,CCOUNT;
```

NQ Profit Function Model – Monte Carlo Simulations (NQSIM.GMS)

```
$OFFLISTING
*------------------------------------------------------------------------
*** NQSIM.GMS ***
*
* VERSION by Hendrik Wolff and Thomas Heckelei:
*
* ------------------------------------------------------------------------

$OFFSYMLIST
$OFFSYMXREF

 OPTION LIMROW  = 0
 OPTION LIMCOL  = 0
 option iterlim =5000

 SETS  I       Production processes         /COT Cotton,WHT Wheat,RI Rice/
       IS(I)   Subset of crops              /COT Cotton,WHT Wheat/
       J       Inputs                       /Land,Capital,Chemical /
       FJ(J)   Fixed inputs                 /Land/
       VJ(J)   Variable inputs              /Capital,Chemical/
       K       Support Points               /1,2/
       G       Observations                 /1*100/
       R(G)    Subset of Observation
       VERS    Versions of different estimation techniques /DATA,ELAS/
       INFO    Information Type of the Results
    /TRUE_A0, TRUE_A1, TRUE_A2, TRUE_A3, TRUE_B1, TRUE_B2, TRUE_B3, TRUE_C1, TRUE_C2, TRUE_C3,
     MEAN_A0, MEAN_A1, MEAN_A2, MEAN_A3, MEAN_B1, MEAN_B2, MEAN_B3, MEAN_C1, MEAN_C2, MEAN_C3,
     BIAS_A0, BIAS_A1, BIAS_A2, BIAS_A3, BIAS_B1, BIAS_B2, BIAS_B3, BIAS_C1, BIAS_C2, BIAS_C3,
     VAR_A0 ,  VAR_A1,  VAR_A2,  VAR_A3,  VAR_B1,  VAR_B2,  VAR_B3,  VAR_C1,  VAR_C2,  VAR_C3,
     RMSE_A0, RMSE_A1, RMSE_A2, RMSE_A3, RMSE_B1, RMSE_B2, RMSE_B3, RMSE_C1, RMSE_C2, RMSE_C3/
    ;

 ALIAS (I,II,S);
 ALIAS (J,JJ);
 ALIAS (FJ,FJ2)

*--- Simulation Steering for the Monte Carlo Experiments -----------------

 SCALAR NSMP       Number of envisaged Monte Carlo Repetitions
/2000/;

 SET GG / 1*6/;
 PARAMETER NOBS(GG) / 1 5, 2 10, 3 20, 4 30, 5 50,6 100/;

* (Note: 1. = 100%)
 SCALAR PERCENT_L  Standard Deviation of Land allocations   /0.05/;

 SCALAR PERCENT_S  Standard Deviation of Supply               /0.1/;

 SCALAR SIGMARULE defines spread around the error            /5/;

*--- Define difference across the R observations -----------------

 SCALAR PERCENT_P  Define std.dev. of output Prices          /0.02/;

 SCALAR PERCENT_W  Define std.dev. of the Input Prices       /0.02/;

DISPLAY '********************** BASIC SETTINGS ***********************************'
DISPLAY NSMP, PERCENT_L, PERCENT_S, PERCENT_P, PERCENT_W;

*------------------------------------------------------------------------
*
*      DATA SET
*
*------------------------------------------------------------------------

 PARAMETER P(I)   observed output prices
     /COT          2.924
      WHT          2.98
      RI           7.09/;

 PARAMETER YB(I)   Average yields
      /COT        220.0
       WHT         85.0
       RI          70.1/;

 PARAMETER B(FJ)   Resource constraints
      /LAND        2.65/;

 PARAMETER W(VJ)    Unit prices of resouWes
       /CAPITAL    10.0
        CHEMICAL   10.0/;
```

```
 TABLE X(I,J)      Observed resource use
                 LAND    CAPITAL   CHEMICAL
       COT       1.49     3.960     2.640
       WHT       0.62     1.980     1.320
       RI        0.54     2.940     1.960
      ;

*-------------------------------------------------------------
*
*       Generate different price vectors
*
*-------------------------------------------------------------

 PARAMETER PG(G,I)  Output Prices for all Observation;
 PARAMETER WG(G,VJ) Input Prices for all Observations;

 PARAMETER NOUT(G,I)        Normalized Output Prices;
 PARAMETER NI(G)            Normalized Input Prices;

*-------------------------------------------------------------
*
*       Define Model to generate data
*      (Parameter taken from the calibration of the Howitt Data as in NQMEELA.GMS)
*
*-------------------------------------------------------------

*
* -------------------------- Set true Parameters of the NQ-Profit Function ----------------------
------------------

*-------------------------------------------------------------
* ALP: in front of linear variables
*-------------------------------------------------------------

 PARAMETER  GALP0(I)    Scale Parameter of the profit function
      /COT -67.6571,    WHT -16.2746,    RI   -8.7735  / ;

 PARAMETER  GALP1(I)    normalized output price Parameter of the profit function
      /COT 115.023,     WHT -34.130,     RI    4.656   /;

 PARAMETER  GALP2(I)    normalized input price Parameter of the profit function
      /COT  1.914,      WHT -0.176,      RI  -0.650 /;

 PARAMETER  GALP3(I,FJ) ressource Parameters of the profit function
      /COT.LAND       87.836
       WHT.LAND       49.817
       RI.LAND        27.335/ ;

*-------------------------------------------------------------
* BETA: in front of quadratic variables
*-------------------------------------------------------------

 PARAMETER  GBETA1(I)   qudratic output price Parameter of the profit function
      /COT 24.854,      WHT 23.552,      RI  6.115/;

 PARAMETER  GBETA2(I)   qudratic input price Parameter of the profit function
      /COT 1.167,       WHT 0.607,       RI  0.882/;

 TABLE GBETA3(I,FJ,FJ2) quadratic ressource Parameters of the profit function
                         LAND
       COT.LAND        -60.321
       WHT.LAND        -82.432
       RI.LAND         -53.635  ;

*-------------------------------------------------------------
* GAMMA: in front of cross products
*-------------------------------------------------------------

 PARAMETER  GGAMMA1(I)  output price by input price Parameter of the profit function
      /COT -9.391,      WHT -4.232,      RI  -2.611/;

 PARAMETER   GGAMMA2(I,FJ)  output price by ressources Parameter of the profit function
      /COT.LAND     144.229
       WHT.LAND     135.553
       RI.LAND       58.284/;

 PARAMETER   GGAMMA3(I,FJ)  input price by ressources Parameter of the profit function
      /COT.LAND     -2.883
       WHT.LAND     -1.855
       RI.LAND      -2.446/;

*-------------------------------------------------------------
*
*       Define Model to generate data
*
*-------------------------------------------------------------
```

```
 VARIABLES  GXF(I,FJ)       allocation of Inputs
            GPROFIT         Total profit

 POSITIVE VARIABLES GXF;

 EQUATIONS
            GRES_           Fixed resources
            GPROFIT_        Profit function
            ;
*
* ---- declare equations -----------------------------------------
*

* --- fixed resource availability

 GRES_(FJ)..    SUM(I, GXF(I,FJ) )  =L= BS(FJ);

* --- Profit Function

 GPROFIT_..  GPROFIT =E= SUM((I,R), GALP0(I) + GALP1(I)*NOUT1(R,I) + GALP2(I)*NOUT1(R)
                           + SUM(FJ,GALP3(I,FJ)*GXF(I,FJ)) + 0.5*GBETA1(I)*SQR(NOUT1(R,I))
                           + 0.5*GBETA2(I)*SQR(NOUT1(R))
                           + 0.5*SUM((FJ,FJ2),GBETA3(I,FJ,FJ2)*GXF(I,FJ)*GXF(I,FJ2))
                           + GGAMMA1(I)*NOUT1(R,I)*NOUT1(R)
                           + SUM(FJ,GGAMMA2(I,FJ)*NOUT1(R,I)*GXF(I,FJ))
                           + SUM(FJ, GGAMMA3(I,FJ)*NOUT1(R)*GXF(I,FJ)));

 MODEL GENNQ /GRES_,GPROFIT_/;

 PARAMETER XOPT(G,I,J)     optimal input allocation
           SEXP(G,I)       expected supply
           LAMOPT(G,FJ)    optimal shadow prices
           SLAM(G,I,FJ)    observed shadow prices
          ;

*---------------------------------------------------------------------
*
*       Define Model for Entropy-Estimation
*
*---------------------------------------------------------------------
 PARAMETER
      SOBS(G,I)        Observed supply
      XOBS(G,I,J)      Observed input allocations
      SPR              Spread for the lower and upper bound for the parameters /2000/
      SPRERRS(I)       spread error supply
      SPRERRX(I,FJ)    spread error input
      SPRELA           spread elasticities
      SUPERRS(I,K)     supports supply errors
      SUPERRX(I,FJ,K)  supports input errors
      SUPELA(I)        supports elasticities
      CENERR           center error supports /0.0/
      CENELA           center error supports
      NOUTSIM(I,S)     normalized output prices for simulation
      NISIM            normalized input price for simulation
          ;

 VARIABLES ENTROPY
      ALP0(I)              Scale Parameter of the profit function
      ALP1(I)              output price Parameter of the profit function
      ALP2(I)              input price Parameter of the profit function
      ALP3(I,FJ)           ressource Parameters of the profit function
      BETA1(I)             quadratic output price Parameter of the profit function
      BETA2(I)             quadratic input price Parameter of the profit function
      BETA3(I,FJ,FJ2)      quadratic ressource Parameters of the profit function
      GAMMA1(I)            output price by input price Parameter of the profit function
      GAMMA2(I,FJ)         output price by ressources Parameter of the profit function
      GAMMA3(I,FJ)         input price by ressources Parameter of the profit function
      LAMBDA(G,FJ)         shadow price (auxiliary variable here - not identified)
      ERRS(G,I)            Disturbance terms supply
      ERRX(G,I,FJ)         Disturbance terms fixed inputs
      ELA (I,II)           Elasticities
      SSIM(I,S)            simulated supply
      XSIM(I,J,S)          simulated input allocations
          ;

 POSITIVE VARIABLES
      PERRS(G,I,K)         Probabilities for supply Error-supports
      PERRX(G,I,FJ,K)      Probabilities for fixed input Error-supports
      PELA(I,K)            Probabilities for Elasticities
          ;
 EQUATIONS
      OBJENT_DATA          Objective Function
      OBJENT_ELA           Objective Function
      SUPPLY_(G,I)         Supply Functions
      SSUPPLY_(G,I,II)     artificial supply functions
      LANDALLO_(G,I,FJ)    Land allocation function (nonlinear)
      DUAL_(G,IS,FJ)       FOC fixed allocable inputs
```

```
      SDUAL_(G,IS,FJ,II)  artificial FOC fixed allocable inputs
      RES_(G,FJ)          resource constraint
      SRES_(G,FJ,II)      artificial resource constraint
      ERRX_(G,I,FJ)       sum over prob * supports for input errors
      ERRS_(G,I)          sum over prob * supports for supply errors
      ELA_(I,II)          sum over prob * supports for elasticities
      SELA_(I,II)         simulated Elasticities
      SUMPERRX_(G,I,FJ)   SUM of Prob. input errors = 1.
      SUMPERRS_(G,I)      SUM of Prob. supply errors = 1.
      SUMPELA_(G,I)       SUM of Prob. elasticities = 1.
;
*
* ---- Entropy Objective Function ----------------------------------------------------------------
------
*
 OBJENT_DATA..
                - SUM((R,I,K),      PERRS(R,I,K)*LOG(PERRS(R,I,K)))
                - SUM((R,I,FJ,K),   PERRX(R,I,FJ,K)*LOG(PERRX(R,I,FJ,K)))
                   =E= ENTROPY;

 OBJENT_ELA..
                - SUM((R,I,K),      PERRS(R,I,K)*LOG(PERRS(R,I,K)))
                - SUM((R,I,FJ,K),   PERRX(R,I,FJ,K)*LOG(PERRX(R,I,FJ,K)))
                - SUM((I,K),        PELA(R,I,FJ,K)*LOG(PELA(R,I,FJ,K)))
                   =E= ENTROPY;

 SUPPLY_(R,I)..      SOBS(R,I) - ERRS(R,I) =E= ALP1(I) + BETA1(I)*NOUT(R,I) + GAMMA1(I)*NI(R)
                                           + SUM(FJ,GAMMA2(I,FJ)*(XOBS(R,I,FJ) - ERRX(R,I,FJ)));


 SSUPPLY_(R,I,II)..  SSIM(I,II) =E= ALP1(I) + BETA1(I)*NOUTSIM(I,II) + GAMMA1(I) * NISIM
                                           + SUM(FJ,GAMMA2(I,FJ)*(XSIM(I,FJ,II)));

*
* ---- Land allocation for each crop (Gyomard model)
*

 LANDALLO_(R,I,FJ)..  XOBS(R,I,FJ) - ERRX(R,I,FJ) =E=
                     (-ALP3(I,FJ) - GAMMA2(I,FJ)*NOUT(R,I) - GAMMA3(I,FJ)*NI(R)
                      + (SUM(II,ALP3(II,FJ)/BETA3(II,FJ,FJ))
                      + (SUM(II,GAMMA2(II,FJ)*NOUT(R,II)/BETA3(II,FJ,FJ)))
                      + (SUM(II,GAMMA3(II,FJ)*NI(R)/BETA3(II,FJ,FJ))) + B(FJ))
                      /  (SUM(II,1/BETA3(II,FJ,FJ)))) / BETA3(I,FJ,FJ);

 DUAL_(R,I,FJ).. LAMBDA(R,FJ) =E= ALP3(IS,FJ)
                                 + SUM(FJ2,BETA3(IS,FJ,FJ2)*(XOBS(R,IS,FJ2) - ERRX(R,IS,FJ2)))
                                 + GAMMA2(IS,FJ)*NOUT(R,IS) + GAMMA3(IS,FJ)*NI(R);

 ERRX_(R,I,FJ)..     ERRX(R,I,FJ) =E= SUM(K,PERRX(R,I,FJ,K)*SUPERRX(I,FJ,K));

 RES_(R,FJ)..        BS(R,FJ) =E= SUM(I, XOBS(R,I,FJ) - ERRX(R,I,FJ));

*
* ---- definition of the error for the supply ----------------------------------------------------
-------
*
 ERRS_(R,I)..        ERRS(R,I)    =E= SUM(K,PERRS(R,I,K)*SUPERRS(I,K));
*
* ---- Summing up of the Probabilities -----------------------------------------------------------
-------

*
 SUMPERRX_(R,I,FJ).. SUM(K,PERRX(R,I,FJ,K)) =E= 1;

 SUMPERRS_(R,I)..    SUM(K,PERRS(R,I,K)) =E= 1;

 MODEL NQTOM_DATA       /OBJENT_DATA, SUPPLY, DUAL_, ERRX_, ERRS_, RES_, SUMPERRX_,
                         SUMPERRS_/;
 MODEL NQTOM_ELA        /OBJENT_ELA , SUPPLY, DUAL_, ERRX_, ERRS_, RES_, SUMPERRX_,
                         SUMPERRS_/;
 MODEL NQGUYME          /OBJENT_,SUPPLY, LANDALLO_, ERRX_, ERRS_,RES_, SUMPERRX_,SUMPERRS_/;

 PARAMETER
        LPED                        Land allocation plus Error hat
        SMPERRS(G,I)                Sampling Errors supply
        SMPERRX(G,I,FJ)             Sampling Errors Land allocation
        MSOBS(I)                    Mean observed supply
        MXOBS(I,FJ)                 Mean observed inputs
        STDVERRS(I)                 Standard deviation supply errors
        STDVERRX(I,FJ)              Standard deviation input errors
        MEANALP0(VERS,GG,I)         Mean of Parameter estimates
        MEANALP1(VERS,GG,I)         Mean of Parameter estimates
        MEANALP2(VERS,GG,I)         Mean of Parameter estimates
        MEANALP3(VERS,GG,I,FJ)      Mean of Parameter estimates
        MEANBETA1(VERS,GG,I)        Mean of Parameter estimates
        MEANBETA2(VERS,GG,I)        Mean of Parameter estimates
        MEANBETA3(VERS,GG,I,FJ,FJ2) Mean of Parameter estimates
```

```
         MEANGAMMA1(VERS,GG,I)         Mean of Parameter estimates
         MEANGAMMA2(VERS,GG,I,FJ)      Mean of Parameter estimates
         MEANGAMMA3(VERS,GG,I,FJ)      Mean of Parameter estimates
         BIASALP0(VERS,GG,I)           BIAS of Parameter estimates
         BIASALP1(VERS,GG,I)           BIAS of Parameter estimates
         BIASALP2(VERS,GG,I)           BIAS of Parameter estimates
         BIASALP3(VERS,GG,I,FJ)        BIAS of Parameter estimates
         BIASBETA1(VERS,GG,I)          BIAS of Parameter estimates
         BIASBETA2(VERS,GG,I)          BIAS of Parameter estimates
         BIASBETA3(VERS,GG,I,FJ,FJ2) BIAS of Parameter estimates
         BIASGAMMA1(VERS,GG,I)         BIAS of Parameter estimates
         BIASGAMMA2(VERS,GG,I,FJ)      BIAS of Parameter estimates
         BIASGAMMA3(VERS,GG,I,FJ)      BIAS of Parameter estimates
         VARALP0(VERS,GG,I)            VAR  of Parameter estimates
         VARALP1(VERS,GG,I)            VAR  of Parameter estimates
         VARALP2(VERS,GG,I)            VAR  of Parameter estimates
         VARALP3(VERS,GG,I,FJ)         VAR  of Parameter estimates
         VARBETA1(VERS,GG,I)           VAR  of Parameter estimates
         VARBETA2(VERS,GG,I)           VAR  of Parameter estimates
         VARBETA3(VERS,GG,I,FJ,FJ2)  VAR  of Parameter estimates
         VARGAMMA1(VERS,GG,I)          VAR  of Parameter estimates
         VARGAMMA2(VERS,GG,I,FJ)       VAR  of Parameter estimates
         VARGAMMA3(VERS,GG,I,FJ)       VAR  of Parameter estimates
         RMSEALP0(VERS,GG,I)           RMSE of Parameter estimates
         RMSEALP1(VERS,GG,I)           RMSE of Parameter estimates
         RMSEALP2(VERS,GG,I)           RMSE of Parameter estimates
         RMSEALP3(VERS,GG,I,FJ)        RMSE of Parameter estimates
         RMSEBETA1(VERS,GG,I)          RMSE of Parameter estimates
         RMSEBETA2(VERS,GG,I)          RMSE of Parameter estimates
         RMSEBETA3(VERS,GG,I,FJ,FJ2) RMSE of Parameter estimates
         RMSEGAMMA1(VERS,GG,I)         RMSE of Parameter estimates
         RMSEGAMMA2(VERS,GG,I,FJ)      RMSE of Parameter estimates
         RMSEGAMMA3(VERS,GG,I,FJ)      RMSE of Parameter estimates
           ;
 PARAMETER COUNT(VERS) Counts the number of optimal MC repetition up to NSMP
             * (but is flexible in order to account for errors)
 PARAMETER CCOUNT(GG,VERS) Counts the number of MC repetition up to NSMP (or less if
                           necessary)
 PARAMETER NINFES(GG,VERS) Counts the number of not optimal solutions;

*****************************************************************************************
*
*          Monte Carlo
*
*****************************************************************************************
*
*  LOOP over the experiments ------------------------------------------------------------
*

LOOP(GG,


* Take the price vectors of appropriate length ------------------------------------------------
---------------

  NOUT(R,I) = PG(R,I) / WG(R,"CHEMICAL");
  NI(R)   = WG(R,"CAPITAL")/WG(R,"CHEMICAL");

  GXF.LO(I,FJ) = 0.00001;
  GXF.L(I,FJ) = X(I,FJ);
  OPTION NLP=CONOPT2;
  GENNQ.Optfile = 3;
  OPTION SOLPRINT = OFF;

* -------------------------------------------------------------------
* Generate observations by using the solver
* (recommended for complex situations, where data cannot be calculated analytically,
*  or equations could get very messy)
* -------------------------------------------------------------------

*LOOP(GG,
*          if ( Sameas(GG,"G4"),
*             R(G)  = No;
*             R(G4) = Yes;
*          );
*          if ( Sameas(GG,"G10"),
*             R(G10) = Yes;
*          );
*          if ( Sameas(GG,"G20"),
*             R(G20) = Yes;
*          );

*LOOP(R,
*

*       SOLVE GENNQ USING NLP MAXIMIZING GPROFIT;
```

```
*       XOPT(R,I,FJ) = GXF.L(I,FJ);

*       LAMOPT(R,FJ) = GRES_.M(FJ);


*
* ---- set start values, upper and lower bounds ------------------------------------------
*
*
* ---- monte carlo loop setup  -----------------------------------------------
*
        MEANALP0(VERS,GG,I)           = 0.;
        MEANALP1(VERS,GG,I)           = 0.;
        MEANALP2(VERS,GG,I)           = 0.;
        MEANALP3(VERS,GG,I,FJ)        = 0.;
        MEANBETA1(VERS,GG,I)          = 0.;
        MEANBETA2(VERS,GG,I)          = 0.;
        MEANBETA3(VERS,GG,I,FJ,FJ2)  = 0.;
        MEANGAMMA1(VERS,GG,I)         = 0.;
        MEANGAMMA2(VERS,GG,I,FJ)      = 0.;
        MEANGAMMA3(VERS,GG,I,FJ)      = 0.;
        VARALP0(VERS,GG,I)            = 0.;
        VARALP1(VERS,GG,I)            = 0.;
        VARALP2(VERS,GG,I)            = 0.;
        VARALP3(VERS,GG,I,FJ)         = 0.;
        VARBETA1(VERS,GG,I)           = 0.;
        VARBETA2(VERS,GG,I)           = 0.;
        VARBETA3(VERS,GG,I,FJ,FJ2)   = 0.;
        VARGAMMA1(VERS,GG,I)          = 0.;
        VARGAMMA2(VERS,GG,I,FJ)       = 0.;
        VARGAMMA3(VERS,GG,I,FJ)       = 0.;
*
* Put errors around the Total expected supply
 STDVERRS(I)    = (SUM(R,SEXP(R,I))/CARD(R)) * PERCENT_S;

*
* Put errors around the land allocations
 STDVERRX(I,FJ) = (SUM(R,XOPT(R,I,FJ))/CARD(R)) * PERCENT_L;

OPTION SOLPRINT = OFF;

 COUNT(VERS)     = 0;
 CCOUNT(GG,VERS)    = 0;
 NINFES(GG,VERS)    = 0;

WHILE( (COUNT("OBS") LT NSMP) AND (COUNT("OPT") LT NSMP) AND (COUNT("OO2") LT NSMP),

DISPLAY NINFES, COUNT, CCOUNT;

 PG(G,I) = P(I)   + NORMAL(0, PERCENT_P * P(I));

 WG(G,VJ) = W(VJ) + NORMAL(0, PERCENT_W * W(VJ));

LOOP(VERS,

  DISPLAY 'DISPLAY just after beginning the VERS LOOP'
  DISPLAY NQTOM_OBSME.modelstat;
  DISPLAY NQTOM_OPTME.modelstat;
  DISPLAY NQTOM_OO2ME.modelstat;
  DISPLAY NINFES, COUNT, CCOUNT;

  PERRX.L(R,I,FJ,K)  = 0.5;
  PERRX.LO(R,I,FJ,K) = 0.0001;
  PERRX.UP(R,I,FJ,K) = 1;

  PERRS.L(R,I,K)     = 0.5;
  PERRS.LO(R,I,K)    = 0.0001;
  PERRS.UP(R,I,K)    = 1;

  ALP0.L(I)            =  GALP0(I);
  ALP1.L(I)            =  GALP1(I);
  ALP2.L(I)            =  GALP2(I);
  ALP3.L(I,FJ)         =  GALP3(I,FJ);
  ALP3.FX("COT",FJ)    =  GALP3("COT",FJ);
  BETA1.L(I)           =  GBETA1(I);
  BETA2.L(I)           =  GBETA2(I);
  BETA3.L(I,FJ,FJ)     =  GBETA3(I,FJ,FJ);
  BETA3.L(I,FJ,FJ2)$(ORD(FJ)ne ord(FJ2))= GBETA3(I,FJ,FJ2);
  GAMMA1.L(I)          =  GGAMMA1(I);
  GAMMA2.L(I,FJ)       =  GGAMMA2(I,FJ);
  GAMMA3.FX("COT",FJ)  =  GGAMMA3("COT",FJ);

  ALP0.Lo(I)           =  GALP0(I) -SPR;          ALP0.UP(I)          = GALP0(I)+SPR;
  ALP1.Lo(I)           =  GALP1(I) -SPR;          ALP1.UP(I)          = GALP1(I)+SPR;
  ALP2.Lo(I)           =  GALP2(I) -SPR;          ALP2.UP(I)          = GALP1(I)+SPR;

  BETA1.Lo(I)          =  +0.0001;                BETA1.UP(I)         = GBETA1(I)+SPR;
```

```
  BETA2.Lo(I)          =  +0.0001;                  BETA2.UP(I)         = GBETA2(I)+SPR;
  BETA3.Lo(I,FJ,FJ)    =  GBETA3(I,FJ,FJ)-SPR;      BETA3.UP(I,FJ,FJ)   = -0.0001;
  BETA3.Lo(I,FJ,FJ2)$(ORD(FJ)ne ord(FJ2)) = GBETA3(I,FJ,FJ2)-SPR;
  BETA3.UP(I,FJ,FJ2)$(ORD(FJ)ne ord(FJ2)) = GBETA3(I,FJ,FJ2)+SPR;

  GAMMA1.Lo(I)         =GGAMMA1(I)-SPR;             GAMMA1.UP(I)         = -0.00001;
  GAMMA2.Lo(I,FJ)      =0.00001;                    GAMMA2.UP(I,FJ)      = GGAMMA2(I,FJ)+SPR;

* Define Errors for the supplies
*
  SMPERRS(R,I)        = NORMAL(0,STDVERRS(I));

* Define Errors for the Land allocations while beeing consistent with the total resource
  constraint B(FJ)
*
  SMPERRX(R,IS,FJ)    = NORMAL(0,STDVERRX(IS,FJ));
  SMPERRX(R,'RI',FJ)  = -SUM(IS,SMPERRX(R,IS,FJ));

* Put Errors around the supplies and around the resource allocations
*
  SOBS(R,I)           = SEXP(R,I) +  SMPERRS(R,I);
  XOBS(R,I,FJ)        = XOPT(R,I,FJ) + SMPERRX(R,I,FJ);
  SPRERRS(I)          = SIGMARULE*STDVERRS(I);
  SPRERRX(I,FJ)       = SIGMARULE*STDVERRX(I,FJ);
  SUPERRS(I,'1')      = CENERR - SPRERRS(I);
  SUPERRS(I,'2')      = CENERR + SPRERRS(I);
  SUPERRX(I,FJ,'1')   = CENERR - SPRERRX(I,FJ);
  SUPERRX(I,FJ,'2')   = CENERR + SPRERRX(I,FJ);
  PERRX.L(R,I,FJ,'1') = (-SMPERRX(R,I,FJ)-SUPERRX(I,FJ,'2'))/(SUPERRX(I,FJ,'1')-
                         SUPERRX(I,FJ,'2'));
  PERRX.L(R,I,FJ,'2') = 1-PERRX.L(R,I,FJ,'1');
  PERRX.LO(R,I,FJ,K) = 0.0001;
  PERRX.UP(R,I,FJ,K) = 1;

  ERRX.L(R,I,FJ) = -SMPERRX(R,I,FJ);
  ERRX.LO(R,I,FJ) = SUPERRX(I,FJ,'1');
  ERRX.UP(R,I,FJ) = SUPERRX(I,FJ,'2');

  PERRS.L(R,I,'1') = (-SMPERRS(R,I)-SUPERRS(I,'2'))/(SUPERRS(I,'1')-SUPERRS(I,'2'));
  PERRS.L(R,I,'2') = 1-PERRS.L(R,I,'1');
  PERRS.LO(R,I,K) = 0.0001;
  PERRS.UP(R,I,K) = 1;

  ERRS.L(R,I) = -SMPERRS(R,I);
  ERRS.LO(R,I) = SUPERRS(I,'1');
  ERRS.UP(R,I) = SUPERRS(I,'2');

  IF (SAMEAS(VERS,"OBS"), SOLVE NQTOM_OBSME USING NLP MAXIMIZING ENTROPY;);
  IF (SAMEAS(VERS,"OPT"), SOLVE NQTOM_OPTME USING NLP MAXIMIZING ENTROPY;);
  IF (SAMEAS(VERS,"OO2"), SOLVE NQTOM_OO2ME USING NLP MAXIMIZING ENTROPY;);

* DISPLAY SMPERRS,SMPERRX,SUPERRS,SUPERRX,SOBS,XOBS,XOPT,SEXP;
* If model is optimal and feasible, calculate means, variances etc....
* Formulas at this point are initialized only. End of calculation comes later - after the while
loop.
* -  1 optimal
* -  2 locally optimal
* -  3 unbounded
* -  4 infeasible
* -  5 locally infeasible
* -  6 intermediate infeasible
* -  7 intermediate non-optimal

IF(    ((SAMEAS(VERS,"OBS")) $ (NQTOM_OBSME.modelstat LE 2))
    OR ((SAMEAS(VERS,"OPT")) $ (NQTOM_OPTME.modelstat LE 2))
    OR ((SAMEAS(VERS,"OO2")) $ (NQTOM_OO2ME.modelstat LE 2)),
*             OR (NQME.modelstat EQ 7)),
        DISPLAY 'DISPLAY in the IF LOOP'
        DISPLAY NQTOM_OBSME.modelstat;
        DISPLAY NQTOM_OPTME.modelstat;
        DISPLAY NQTOM_OO2ME.modelstat;
        DISPLAY NINFES, COUNT, CCOUNT;
        MEANALP0(VERS,GG,I)           = MEANALP0(VERS,GG,I)         +ALP0.L(I)         ;
        MEANALP1(VERS,GG,I)           = MEANALP1(VERS,GG,I)         +ALP1.L(I)         ;
        MEANALP2(VERS,GG,I)           = MEANALP2(VERS,GG,I)         +ALP2.L(I)         ;
        MEANALP3(VERS,GG,I,FJ)        = MEANALP3(VERS,GG,I,FJ)      +ALP3.L(I,FJ)      ;
        MEANBETA1(VERS,GG,I)          = MEANBETA1(VERS,GG,I)        +BETA1.L(I)        ;
        MEANBETA2(VERS,GG,I)          = MEANBETA2(VERS,GG,I)        +BETA2.L(I)        ;
        MEANBETA3(VERS,GG,I,FJ,FJ2)   = MEANBETA3(VERS,GG,I,FJ,FJ2)+BETA3.L(I,FJ,FJ2);
        MEANGAMMA1(VERS,GG,I)         = MEANGAMMA1(VERS,GG,I)       +GAMMA1.L(I)       ;
        MEANGAMMA2(VERS,GG,I,FJ)      = MEANGAMMA2(VERS,GG,I,FJ)    +GAMMA2.L(I,FJ)    ;
        MEANGAMMA3(VERS,GG,I,FJ)      = MEANGAMMA3(VERS,GG,I,FJ)    +GAMMA3.L(I,FJ)    ;

        VARALP0(VERS,GG,I)            = VARALP0(VERS,GG,I) +ALP0.L(I)   * ALP0.L(I)    ;
        VARALP1(VERS,GG,I)            = VARALP1(VERS,GG,I) +ALP1.L(I)   * ALP1.L(I)    ;
        VARALP2(VERS,GG,I)            = VARALP2(VERS,GG,I) +ALP2.L(I)   * ALP2.L(I)    ;
        VARALP3(VERS,GG,I,FJ)         = VARALP3(VERS,GG,I,FJ) +ALP3.L(I,FJ) * ALP3.L(I,FJ);
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```
        VARBETA1(VERS,GG,I)           = VARBETA1(VERS,GG,I) +BETA1.L(I) * BETA1.L(I)      ;
        VARBETA2(VERS,GG,I)           = VARBETA2(VERS,GG,I) +BETA2.L(I) * BETA2.L(I)      ;
        VARBETA3(VERS,GG,I,FJ,FJ2)    =  VARBETA3(VERS,GG,I,FJ,FJ2) +BETA3.L(I,FJ,FJ2)*
                                         BETA3.L(I,FJ,FJ2);
        VARGAMMA1(VERS,GG,I)          = VARGAMMA1(VERS,GG,I)  +GAMMA1.L(I) * GAMMA1.L(I)  ;
        VARGAMMA2(VERS,GG,I,FJ)       = VARGAMMA2(VERS,GG,I,FJ) +GAMMA2.L(I,FJ) *
                                         GAMMA2.L(I,FJ)    ;
        VARGAMMA3(VERS,GG,I,FJ)       = VARGAMMA3(VERS,GG,I,FJ)     +GAMMA3.L(I,FJ)    *
                                         GAMMA3.L(I,FJ)    ;
    COUNT(VERS)  = COUNT(VERS) + 1;
    CCOUNT(GG,VERS) = CCOUNT(GG,VERS) + 1;
     ELSE
        NINFES(GG,VERS) = NINFES(GG,VERS) + 1;
        DISPLAY 'DISPLAY in the ELSE LOOP'
        DISPLAY NQTOM_OBSME.modelstat;
        DISPLAY NQTOM_OPTME.modelstat;
        DISPLAY NQTOM_OO2ME.modelstat;
        DISPLAY NINFES, COUNT,CCOUNT;
* end of If/Else - Loop
   );
* end of LOOP (VERS,
 );
* end of While-Loop
);
*
* Calculation of Means, Variances, Bias and RMSE
*
* In order to avoid division by zero, set CCOUNT = 0 to CCOUNT = 1
 LOOP (VERS, IF (CCOUNT(GG,VERS) EQ 0, CCOUNT(GG,VERS) = 1));

 MEANALP0(VERS,GG,I)          = MEANALP0(VERS,GG,I)          /CCOUNT(GG,VERS);
 MEANALP1(VERS,GG,I)          = MEANALP1(VERS,GG,I)          /CCOUNT(GG,VERS);
 MEANALP2(VERS,GG,I)          = MEANALP2(VERS,GG,I)          /CCOUNT(GG,VERS);
 MEANALP3(VERS,GG,I,FJ)       = MEANALP3(VERS,GG,I,FJ)       /CCOUNT(GG,VERS);
 MEANBETA1(VERS,GG,I)         = MEANBETA1(VERS,GG,I)         /CCOUNT(GG,VERS);
 MEANBETA2(VERS,GG,I)         = MEANBETA2(VERS,GG,I)         /CCOUNT(GG,VERS);
 MEANBETA3(VERS,GG,I,FJ,FJ2)= MEANBETA3(VERS,GG,I,FJ,FJ2)/CCOUNT(GG,VERS);
 MEANGAMMA1(VERS,GG,I)        = MEANGAMMA1(VERS,GG,I)        /CCOUNT(GG,VERS);
 MEANGAMMA2(VERS,GG,I,FJ)     = MEANGAMMA2(VERS,GG,I,FJ)     /CCOUNT(GG,VERS);
 MEANGAMMA3(VERS,GG,I,FJ)     = MEANGAMMA3(VERS,GG,I,FJ)     /CCOUNT(GG,VERS);

 BIASALP0(VERS,GG,I)          = MEANALP0(VERS,GG,I)          -GALP0(I)          ;
 BIASALP1(VERS,GG,I)          = MEANALP1(VERS,GG,I)          -GALP1(I)          ;
 BIASALP2(VERS,GG,I)          = MEANALP2(VERS,GG,I)          -GALP2(I)          ;
 BIASALP3(VERS,GG,I,FJ)       = MEANALP3(VERS,GG,I,FJ)       -GALP3(I,FJ)       ;
 BIASBETA1(VERS,GG,I)         = MEANBETA1(VERS,GG,I)         -GBETA1(I)         ;
 BIASBETA2(VERS,GG,I)         = MEANBETA2(VERS,GG,I)         -GBETA2(I)         ;
 BIASBETA3(VERS,GG,I,FJ,FJ2)= MEANBETA3(VERS,GG,I,FJ,FJ2)-GBETA3(I,FJ,FJ2);
 BIASGAMMA1(VERS,GG,I)        = MEANGAMMA1(VERS,GG,I)        -GGAMMA1(I)        ;
 BIASGAMMA2(VERS,GG,I,FJ)     = MEANGAMMA2(VERS,GG,I,FJ)     -GGAMMA2(I,FJ)     ;
 BIASGAMMA3(VERS,GG,I,FJ)     = MEANGAMMA3(VERS,GG,I,FJ)     -GGAMMA3(I,FJ)     ;

 VARALP0(VERS,GG,I)          = VARALP0(VERS,GG,I)          /CCOUNT(GG,VERS) - MEANALP0(VERS,GG,I)
* MEANALP0(VERS,GG,I)          ;
 VARALP1(VERS,GG,I)          = VARALP1(VERS,GG,I)          /CCOUNT(GG,VERS) - MEANALP1(VERS,GG,I)
* MEANALP1(VERS,GG,I)          ;
 VARALP2(VERS,GG,I)          = VARALP2(VERS,GG,I)          /CCOUNT(GG,VERS) - MEANALP2(VERS,GG,I)
* MEANALP2(VERS,GG,I)          ;
 VARALP3(VERS,GG,I,FJ)       = VARALP3(VERS,GG,I,FJ)       /CCOUNT(GG,VERS) -
MEANALP3(VERS,GG,I,FJ)       * MEANALP3(VERS,GG,I,FJ)       ;
 VARBETA1(VERS,GG,I)         = VARBETA1(VERS,GG,I)         /CCOUNT(GG,VERS) - MEANBETA1(VERS,GG,I)
* MEANBETA1(VERS,GG,I)         ;
 VARBETA2(VERS,GG,I)         = VARBETA2(VERS,GG,I)         /CCOUNT(GG,VERS) - MEANBETA2(VERS,GG,I)
* MEANBETA2(VERS,GG,I)         ;
 VARBETA3(VERS,GG,I,FJ,FJ2)= VARBETA3(VERS,GG,I,FJ,FJ2)/CCOUNT(GG,VERS) -
MEANBETA3(VERS,GG,I,FJ,FJ2)* MEANBETA3(VERS,GG,I,FJ,FJ2);
 VARGAMMA1(VERS,GG,I)        = VARGAMMA1(VERS,GG,I)        /CCOUNT(GG,VERS) - MEANGAMMA1(VERS,GG,I)
* MEANGAMMA1(VERS,GG,I)        ;
 VARGAMMA2(VERS,GG,I,FJ)     = VARGAMMA2(VERS,GG,I,FJ)     /CCOUNT(GG,VERS) -
MEANGAMMA2(VERS,GG,I,FJ)     * MEANGAMMA2(VERS,GG,I,FJ)     ;
 VARGAMMA3(VERS,GG,I,FJ)     = VARGAMMA3(VERS,GG,I,FJ)     /CCOUNT(GG,VERS) -
MEANGAMMA3(VERS,GG,I,FJ)     * MEANGAMMA3(VERS,GG,I,FJ)     ;

 RMSEALP0(VERS,GG,I)          = SQRT(SQRT(SQR(VARALP0(VERS,GG,I)          + BIASALP0(VERS,GG,I)
* BIASALP0(VERS,GG,I)          )));
 RMSEALP1(VERS,GG,I)          = SQRT(SQRT(SQR(VARALP1(VERS,GG,I)          + BIASALP1(VERS,GG,I)
* BIASALP1(VERS,GG,I)          )));
 RMSEALP2(VERS,GG,I)          = SQRT(SQRT(SQR(VARALP2(VERS,GG,I)          + BIASALP2(VERS,GG,I)
* BIASALP2(VERS,GG,I)          )));
 RMSEALP3(VERS,GG,I,FJ)       = SQRT(SQRT(SQR(VARALP3(VERS,GG,I,FJ)       + BIASALP3(VERS,GG,I,FJ)
* BIASALP3(VERS,GG,I,FJ)       )));
 RMSEBETA1(VERS,GG,I)         = SQRT(SQRT(SQR(VARBETA1(VERS,GG,I)         + BIASBETA1(VERS,GG,I)
* BIASBETA1(VERS,GG,I)         )));
 RMSEBETA2(VERS,GG,I)         = SQRT(SQRT(SQR(VARBETA2(VERS,GG,I)         + BIASBETA2(VERS,GG,I)
* BIASBETA2(VERS,GG,I)         )));
 RMSEBETA3(VERS,GG,I,FJ,FJ2)= SQRT(SQRT(SQR(VARBETA3(VERS,GG,I,FJ,FJ2)+
BIASBETA3(VERS,GG,I,FJ,FJ2)* BIASBETA3(VERS,GG,I,FJ,FJ2))));
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 RMSEGAMMA1(VERS,GG,I)        = SQRT(SQRT(SQR(VARGAMMA1(VERS,GG,I)       + BIASGAMMA1(VERS,GG,I)
* BIASGAMMA1(VERS,GG,I)        )));
 RMSEGAMMA2(VERS,GG,I,FJ)     = SQRT(SQRT(SQR(VARGAMMA2(VERS,GG,I,FJ)    +
BIASGAMMA2(VERS,GG,I,FJ)     * BIASGAMMA2(VERS,GG,I,FJ)    )));
 RMSEGAMMA3(VERS,GG,I,FJ)     = SQRT(SQRT(SQR(VARGAMMA3(VERS,GG,I,FJ)    +
BIASGAMMA3(VERS,GG,I,FJ)     * BIASGAMMA3(VERS,GG,I,FJ)    )));

DISPLAY COUNT,CCOUNT,NINFES,NSMP;
* end of loop GG
);
DISPLAY
MEANALP0,MEANALP1,MEANALP2,MEANALP3,MEANBETA1,MEANBETA2,MEANBETA3,MEANGAMMA1,MEANGAMMA2,MEANGAMMA
3,
BIASALP0,BIASALP1,BIASALP2,BIASALP3,BIASBETA1,BIASBETA2,BIASBETA3,BIASGAMMA1,BIASGAMMA2,BIASGAMMA
3,
VARALP0,VARALP1,VARALP2,VARALP3,VARBETA1,VARBETA2,VARBETA3,VARGAMMA1,VARGAMMA2,VARGAMMA3;
DISPLAY RMSEALP0,RMSEALP1,RMSEALP2,RMSEALP3,
RMSEBETA1,RMSEBETA2,RMSEBETA3,RMSEGAMMA1,RMSEGAMMA2,RMSEGAMMA3;

PARAMETER SNRMSE(VERS,GG) Sum of normalized RMSE;
PARAMETER SRMSE(VERS,GG)  Sum of RMSE;

 SNRMSE(VERS,GG)= SUM(I,RMSEALP0(VERS,GG,I)                 /ABS(GALP0(I)                 ))
           + SUM(I,RMSEALP1(VERS,GG,I)               /ABS(GALP1(I)               ))
           + SUM(I,RMSEALP2(VERS,GG,I)               /ABS(GALP2(I)               ))
           + SUM(I,RMSEALP3(VERS,GG,I,"LAND")        /ABS(GALP3(I,"LAND")        ))
           + SUM(I,RMSEBETA1(VERS,GG,I)              /ABS(GBETA1(I)              ))
           + SUM(I,RMSEBETA2(VERS,GG,I)              /ABS(GBETA2(I)              ))
           + SUM(I,RMSEBETA3(VERS,GG,I,"LAND","LAND")/ABS(GBETA3(I,"LAND","LAND")))
           + SUM(I,RMSEGAMMA1(VERS,GG,I)             /ABS(GGAMMA1(I)             ))
           + SUM(I,RMSEGAMMA2(VERS,GG,I,"LAND")      /ABS(GGAMMA2(I,"LAND")      ))
           + SUM(I,RMSEGAMMA3(VERS,GG,I,"LAND")      /ABS(GGAMMA3(I,"LAND")      ));

 SRMSE(VERS,GG) = SUM(I,RMSEALP0(VERS,GG,I)                )
           + SUM(I,RMSEALP1(VERS,GG,I)               )
           + SUM(I,RMSEALP2(VERS,GG,I)               )
           + SUM(I,RMSEALP3(VERS,GG,I,"LAND")        )
           + SUM(I,RMSEBETA1(VERS,GG,I)              )
           + SUM(I,RMSEBETA2(VERS,GG,I)              )
           + SUM(I,RMSEBETA3(VERS,GG,I,"LAND","LAND"))
           + SUM(I,RMSEGAMMA1(VERS,GG,I)             )
           + SUM(I,RMSEGAMMA2(VERS,GG,I,"LAND")      )
           + SUM(I,RMSEGAMMA3(VERS,GG,I,"LAND")      );

DISPLAY SNRMSE,SRMSE;

PARAMETER RESULT(*,VERS,GG,I);

RESULT("TRUE_A0",VERS,GG,I)= GALP0(I)                ;
RESULT("TRUE_A1",VERS,GG,I)= GALP1(I)                ;
RESULT("TRUE_A2",VERS,GG,I)= GALP2(I)                ;
RESULT("TRUE_A3",VERS,GG,I)= GALP3(I,"LAND")         ;
RESULT("TRUE_B1",VERS,GG,I)= GBETA1(I)               ;
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4 Estimated Programming Models within the Large Scale Policy Information System CAPRI

4.1 Introduction to Chapter 4

This chapter puts the estimation of agricultural programming models into the context of the large scale agricultural information system 'Common Agricultural Policy Regional Impact' (CAPRI), which was conceptualised and implemented as a tool for policy analysis during the years 1997 to 1999 and which is currently updated and revised in a follow-up project financed by the European Commission (CAP-STRAT). The author was a co-leader of the project's co-ordinating team from the University of Bonn, responsible for the methodological concept and the development of a European research network.

The chapter is structured as follows: the next section presents the general concept of CAPRI and the results of an explorative application to evaluate policy changes introduced by the 'Agenda 2000'. It is largely based on HECKELEI and BRITZ (2001) and sets the general framework for the use of programming models within such an information system. The subsequent section motivates in detail the employed crop supply specification and largely refers to HECKELEI and BRITZ (2000). The underlying estimation of regional cost functions is the first one to use multiple observations in an econometric type approach, but it is still based on the PMP procedure employing dual values of calibration constraints from the 'first phase'. Therefore, a critical evaluation in the light of findings from chapter two and three follows in section four which lead to an outlook to methodological modifications envisaged for the current update of the modelling system. Section five summarises.

4.2 Concept and Explorative Application of CAPRI

4.2.1 Motivation and Overview

The main objective of the project CAPRI (**C**ommon **A**gricultural **P**olicy **R**egional **I**mpact) was the development of an EU-wide economic modelling system able to analyse the regional impacts of the Common Agricultural Policy (CAP). The project was co-financed by EU under the FAIR program in the years 1997-1999.

In order to achieve its ambitious objective, the project relied on the functionality of a European research network. Each of the five main partners[39] was responsible for a specific cluster of Member States. They established research relationships with national sub-partners for data collection and interpretation of results. We introduce the concept and implementation of the CAPRI *database* (subsection 2) and *modelling system* (subsection 3). Selected results of an explorative application to a reference run and an Agenda 2000 scenario are presented to illustrate the type of information which can be generated with the modelling system (subsection 4). A short outlook on current general model developments is given in subsection 5.[40]

4.2.2 CAPRI Database

A major part of the CAPRI project was devoted to sample data and compile the regionalised CAPRI database. The involvement of different teams, the necessity to create a EU-wide comparable information base, and the requirements of the economic modelling system demands a well defined database. The CAPRI database obeys the following principles:

- *Regional differentiation* of the European Union to 200 regional units (mostly according to NUTS II definition[41])
- *Production activity based break-down of agricultural production and input use*
- *Consistency* between sectoral and regional aggregates, i.e. data match official Eurostat statistics including the Economic Accounts of Agriculture (EAA)
- *Comprehensiveness:* complete coverage of product generation and input use according to the EAA, inclusion of activity levels, yields, input coefficients, prices, farm & market balances, economic performance, political instruments and environmental indicators

Currently, the database is complete for the years 1990-1995 for all regions. The key concept of the CAPRI database is termed Activity Based Accounting System (ABAS, WOLF 1995) breaking down the agricultural production process for each period to individual production and use activities, both in physical and valued terms. CAPRI differentiates between 60 outputs and 35

[39] The research teams involved are institutes in the field of agricultural economics from the Universities of Bonn, Valencia, Galway, Bologna, and Montpellier (plus Research Station Tänikon (Switzerland) and NILF, Oslo).

[40] This presentation gives only a very limited account of the research performed within the CAPRI-project. The CAPRI web site (http://www.agp.uni-bonn.de/agpo/rsrch/capri/capri_e.htm) and the final report of the CAPRI project (CAPRI 2000) provide further information.

[41] The Nomenclature of territorial units for statistics (NUTS) of the EU is a five-level hierarchical classification (three regional levels and two local levels). The NUTS subdivides each Member State into a whole number of NUTS 1 regions, each of which is in turn subdivided into a whole number of NUTS 2 regions and so on.

inputs, covering the whole agricultural sector according to EAA definitions, and about 50 crop and animal production activities.

The output and input coefficients are defined consistently to sectoral output generation and input use (see Figure 4.1). *Use activities* which define so called 'farm balances' for each output and input describe the fate of the outputs and input 'generation'. Output produced may be sold, added to stocks, fed, used as seed etc. Inputs may be bought, taken out of stocks or originate from intra-sectoral transactions, for example young animals may be produced by another production activity. In order to link the physical sphere with the EAA, national *unit value prices* are used. They are residually defined by definitorial equations underlying the methodology of the EAA.

Figure 4.1: Activity Based Accounting System (ABAS)

Physical Component			Price Component	Valued Component (Economic Accounts of Agricultural - EAA in gross and net concept)				
O-coefficients (x activity levels = output generation)	Farm balances for outputs (output use)	X	Output Prices	=	Gross Output	- Intermediate Output use	=	Net Output (EAA)
I-coefficients (x activity levels = input generation)	Farm balances for inputs (input use)	X	Input Prices	=	Gross Input	- Intermediate Input use	=	Net Input (EAA)
Income indicators per activity					Sectoral income indicators			Income (EAA)

At national level, the project relies to a greater extent on the SPEL-EU database (WOLF 1995) from the Statistical Office of the European Communities (Eurostat), which integrates different databases, technological information and expert knowledge, and covers longer time series for all EU Member States. The REGIO domain of Eurostat represents the uniform regional data source which suffers, however, from incompleteness and a partially insufficient level of differentiation. Completely missing is information on CAP measures at regional level. Consequently, many statistical sources at national and even regional level had to be found, accessed, analysed, and compiled to achieve a uniform and complete database. The two key factor of success for this enormous task were (1) the establishment of a network of researchers from all Member States and (2) a clear methodological concept.

The database also comprises a set of *environmental indicators*. Useful indicators at this stage of the CAPRI information system are defined by (1) a direct link to the agricultural production

system, (2) meaningful interpretation at CAPRI's current regional level of differentiation, i.e. the NUTS II level, and (3) being operational with respect to data availability. These definitions exclude indicators which describe states of environmental problems at local level or with respect to ecological systems defined by specific regional boundaries (e.g. water catching, landscape). CAPRI, however, offers the unique chance to apply appropriate indicators in a consistent and uniform manner across Europe relating to the regional agricultural production system. Based on these considerations the project implemented nutrient balances and gas emissions relevant for global climatic change for all regions in the system.

4.2.3 Modelling System

Overall Concept

From a methodological point of view, the main challenge was the development of a modelling system which could combine deep regionalisation with complete coverage of the EU-agricultural sector. This set-up was necessary in order to simultaneously analyse the effect of commodity market and policy developments on agriculture in the individual regions as well as the feedback from the regions to EU and world markets.

Since market and activity specific policy instruments require a rather disaggregated model in terms of products, a simultaneous system which would optimise producer and consumer surplus to simulate competitive markets for 200 regions and some 50 products was computationally infeasible. Consequently, the model system was conceptually split-up into a supply and a market component. The supply module consists of individual programming models for about 200 NUTS II regions. The market module follows the tradition of multi-commodity models. Based on aggregated supply quantities from the regional models, the market model returns market clearing prices. An iterative process between the supply and market component ultimately achieves a comparative static equilibrium.

Supply module

The supply module consists of independent regional programming models, well-suited for a high degree of activity differentiation and the direct representation of relevant farm policy measures (e.g. premiums, set-aside obligations) and ensures simulation results consistent with general resource constraints. The objective functions maximise the aggregated gross value added including CAP premiums minus a quadratic cost function based on PMP.

The choice of the optimal production mix is restricted by a relative small number of constraints: availability of arable and permanent grass land, quotas on sales for milk and sugar beets, set-aside obligations, base area related premium reductions, and upper bounds for voluntary set-aside according to CAP regulations. Feed costs are minimised endogenously by determining the optimal mix of a limited number of aggregated marketable (e.g. 'cereals') and non-tradable feedingstuffs (e.g. 'hay') subject to requirement constraints, ensuring a technologically plausible mix. Nutrient requirements of crops can be covered either by mineral or organic fertilisers, the latter restricted to the amount produced by the regional herds. Constraints ensure that a crop specific percentage of the nutrient need is covered by mineral fertiliser.

In order to obtain a plausible, i.e. non specialised and smooth, aggregate supply response for the regional programming models, non-linear cost terms were introduced to the objective functions. The specification of these cost functions generally follows the idea of PMP, but for the first time employs a cross sectional estimation procedure based on multiple observations (HECKELEI and BRITZ 2000). Motivation and details of this approach are given in the next main section of this chapter.

Market Module

Methodological solutions for the market module are generally based on a the standard concept of multi-commodity models (BRITZ 1998). Double log equations for supply and demand clear regional and international markets, driven by regional producer and consumer prices which are linked via price transmission functions to a uniform world market price. The parameters of the behavioural demand equations are not estimated, but instead calibrated under theoretical restrictions based on elasticity estimates taken from literature (WITZKE and BRITZ 1998).

The non-spatial net-trade model is regionalised at EU Member State level, Switzerland, Norway, and 'Rest-of-the-World' (ROW). Data, behavioural parameters and exogenous shifts for ROW come to a large extent from the Word Agricultural Trade Simulation System (WATSIM), a world wide modelling system for trade in agricultural products (VON LAMPE 1998). Supply for all other regions is fixed to the results of the regional supply models. Price transmission functions cover tariffs, including flexible levies depending on internal price floors, as well as marketing and processing costs.

Processing of oilseeds is modelled explicitly assuming fixed extraction rates for cakes and oils from crushing. In the case of processed milk products (skimmed milk powder, butter and other), constraints equilibrate the fat and protein content of processed quantities of raw milk with the

processed products. The price of raw milk and processed milk products is derived from uniform fat and protein prices weighted with their contents plus fixed per unit processing costs.

4.2.4 Application

The CAPRI modelling system was tested in late 1999 in an Agenda 2000 scenario (simulation run) compared to a continuation of the status quo policy for the European agricultural sector (reference run) for the year 2005. The subsequent description of scenario definition and selected model results is restricted to 'cereals and oilseeds' and the 'beef and dairy' sector, as the main target sectors of Agenda 2000.

Cereals and Oilseeds Sector

The political instruments of both scenarios for cereals and oilseeds are presented in the following table. Note that premiums in Table 4.1 represent averages weighted by observed (base year) and projected (reference and Agenda scenario) regional activity levels so that the resulting values are partly endogenous. Furthermore, differences between base year and reference scenario are also caused by (1) the third step of the 92 CAP reform which was not fully implemented in the base year and (2) a set of already decided changes such as the adjustment of historical yields for some regions.

Table 4.1: Political Variables for Cereals and Oilseeds

	Base year 1994	Reference 2005 (% change to base year)	Agenda 2005 (% change to reference)
Cereals			
Intervention price	143.5	123,0 (-14%)	104,6 (-15%)
Average premium per ha	211.6	274,4 (30%)	319,9 (17%)
Oilseeds			
Average premium per ha	449.8	443,9 (-1%)	275,3 (-38%)
Set aside			
Set aside rate (in %)	14.0	17.5	10.0
Average premium per ha	289.0	315,0 (9%)	303,4 (-4%)

Source: Own calculations based on CAPRI data base and simulation runs.

Additionally, the following assumptions apply to the scenario definitions:

- + 1.33% yield increase per year for cereals (EU average, regionalised at national level); + 1.45% yield increase per year for oilseeds (EU average, regionalised at national level); (inputs adjusted accordingly with input saving technical progress of 0.5% per year)
- All oilseeds are cultivated under the main scheme (i.e. receive premiums)
- Small producer share is kept constant at base year levels.

Table 4.2: Activity Levels, Grandes Cultures (in 1000 hectares)

	Base year 1994	Reference 2005	Agenda 2005	Reference to Base year	Agenda to Reference
Cereals (excl. rice)	35012	32663	33796	-6.7%	3.5%
Wheat	16018	14990	15230	-6.4%	1.6%
Barley	11072	10352	10916	-6.5%	5.4%
Other cereals	7922	7321	7650	-7.6%	4.5%
Pulses	1680	1676	1699	-0.2%	1.4%
Oilseeds	5273	5141	4848	-2.5%	-5.7%
Rapeseed	2258	2359	2165	4.5%	-8.2%
Sunflower seed	2740	2512	2415	-8.3%	-3.9%
Soya beans	274	270	269	-1.6%	-0.4%
Non Food on set aside	618	935	777	51.3%	-16.9%
Set aside	4131	5468	4063	32.4%	-25.7%

Source: CAPRI database and own calculcations.

At first we want to have a short look at aggregated EU-results. Table 4.2 presents activity levels for Grandes Cultures. The main developments from the *base year to the reference scenario* include a decrease of the cereal area by 6.7% to 32.7 million ha, mainly due to the increased set-aside rate. However, with technical progress driving up yields, the production is estimated to increase by 9.3% to about 192 million tons. Table 4.3 allows a differentiated look at market effects for wheat and barley. As domestic demand is nearly unchanged, net exports and/or intervention sales expand. Intervention prices for both cereals exceed simulated world market prices, as in many studies.

Table 4.3: Balance Sheet Cereals (in 1000 tons)

	Base year 1994	Reference 2005	Agenda 2005	Reference to Base year	Agenda to Reference
Wheat					
Domestic supply	81659	90325	91134	10.6%	0.9%
Domestic demand	67232	66676	66149	-0.8%	-0.8%
Feed use	30041	29499	28948	-1.8%	-1.9%
Intervention	7799	8348	0	7.0%	-100.0%
Barley					
Domestic supply	42468	46150	47752	8.7%	3.5%
Domestic demand	29250	29719	30129	1.6%	1.4%
Feed use	29026	29497	29906	1.6%	1.4%
Intervention	4469	6430	7624	43.9%	18.6%

Source: CAPRI database and own calculations.

Comparing *Agenda 2000 results to the reference run*, reduced set-aside rates increase cereal production by 3.5% to 195.8 million tons. Lower prices cause slightly extended domestic use, but do not affect EU's status as a cereal net exporter. In opposite to the reference scenario, world market prices for wheat are simulated to lie above intervention price level, allowing wheat exports without subsidies and WTO restrictions. Consequently, net exports rise considerably (Table 4.3). However, the simulated difference between world market and intervention price is rather small. With respect to barley, the intervention price is still above world market price

implying continuing problems since exports would require subsidies and are limited by WTO restrictions.

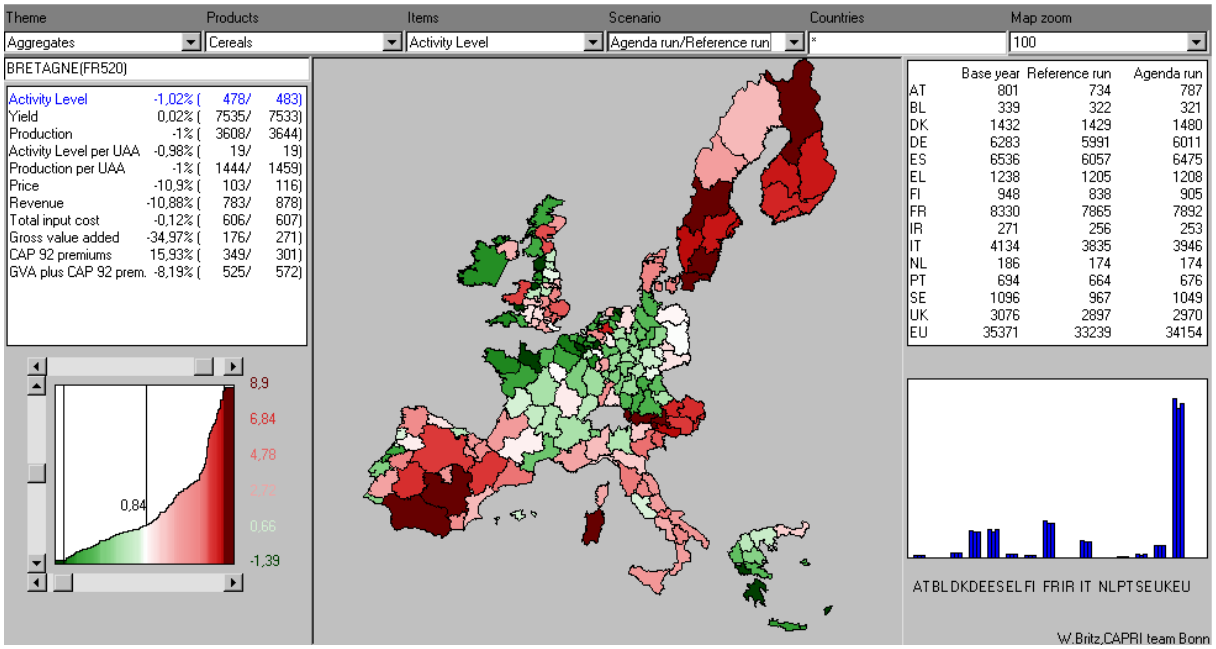
Production of oilseeds is simulated to expand as well in the reference compared to the base year despite a slight area reduction. In the Agenda 2000 scenario oilseed premiums decrease quite drastically to the level of cereal premiums. The loss of profitability results in an estimated 6% reduction in oilseeds areas compared to reference run results (Table 4.2). Compared to the drastic premium cut, the simulation response of the model may at first look small. However, the following aspects should be taken into account:

- The Blair House agreement is no longer in effect. Consequently, the drop of the effective set-aside rate for oilseeds is larger than for cereals in many regions.
- Effective oilseeds premiums in the reference run are reduced in several Member States due to a simulated EU wide 8% overshoot of base areas, so that a simple comparison between declared oilseed premiums before and after Agenda 2000 is misleading.
- Sunflower seeds are much more resistant to droughts, so that a substitution with cereals in southern regions is restricted by availability of irrigation.

The aggregated results are already influenced by the model's capability to represent policy implementation at regional level which eliminates part of the aggregation error of aggregated models. Now we want have a closer look at some regional aspects of the policy impacts. Figure 4.2 shows the effect of Agenda 2000 on regional cereal activity levels. Compared to the reference run cereal area increases in most regions of Spain, Italy, Austria, east England, and the Scandinavian countries. It remains rather constant in the main cereal producing regions of France and Germany.

One reason for the differences are increased reference yields implying higher area premiums for cereals in some Spanish and Italian regions due to a special agreement in Agenda 2000. Premiums in Spain increase by about 27%, in Italy by about 20% and in the rest of Europe by 16%. Another reason is the lower price reduction for maize and durum wheat. Whereas the intervention price for cereals falls by 15% (from Reference to Agenda) the price for durum wheat falls by 13.7% and for maize by only 2.6%. Both crops (especially durum wheat) are primarily grown in southern Europe.

Figure 4.2: Impacts of Agenda 2000 on Cereal Production



Source: CAPRI database and own calculations.

Due to the premium effect of adjusted historic yields the reduction of set-aside in Mediterranean regions is generally in line with the changes of the official set-aside rate. In the highly productive cereal regions of northern France and Germany, however, obligatory set-aside reduction is partially compensated by increased voluntary set-aside, because the under-compensation of the overall price cut in Agenda 2000 (only about 50% based on historic yields) diminishes gross value added considerably. This effective under-compensation in 2005 is even stronger in regions with high technical progress in cereal production, namely French and German regions.

Beef and Dairy Sector

As the main political instruments of both scenarios have been widely discussed, the following Table 4.4 just presents average changes of the quantitative measures in the sector at EU level.

Table 4.4: Political Variables for the Cattle Sector

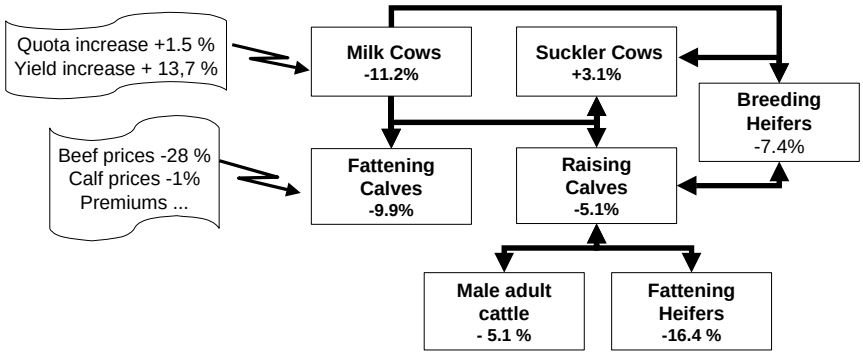
	Base year 1994	Reference 2005 (% change to base year)	Agenda 2005 (% change to reference)
Administrative prices			
Beef	4285	3475 (-19%)	2780 (-20%)
Butter	3202	2954 (-8%)	2511 (-15%)
Milk powder	2377	2055 (-14%)	1747 (-15%)
Premiums			
Milk cows	0	0	157
Suckler cows	133	164 (23%)	284 (73%)
Male adult cattle	105	136 (30%)	324 (138%)
Milk quota	113879	115577	120335

Source: CAPRI database and own calculations.

The following assumptions also apply:

- Milk yields increase due to technical progress (specific trend for each Member State)
- Long term trend to increased final weights continues to offset reduced availability of calves: + 10% for male adult cattle and heifers fattening are assumed until 2005
- Adjustments of feed requirements according to yield development (milk, final weight) but 0.5% increase in feed efficiency per year.

Figure 4.3: Development of Herd Sizes in the EU in Reference Run



Source: Own calculations.

Under both scenarios (reference run and Agenda 2000), the production of milk is clearly quota driven. Whereas production slightly increases following the quota expansion, the increase of average milk yields per cow leads to a distinct reduction of the dairy cow herd in Europe, affecting other cattle activities due to reduced output of calves as well as decreased demand for young cows.

Figure 4.3 shows the results from the reference run: the dairy cow herd is more or less exogenously determined by the slight quota increase (+1.5%) combined with an average increase of milk yields by about 14%. The reduced availability of calves keeps their prices relatively stable despite a drop of the beef price by 28%. However, part of the price drop is compensated increasing premiums for male adult cattle. The stable calve prices favour suckler cows (herd size increases by 3.5%). Reduced availability of calves decreases fattening of heifers (-17%) and calves (-10%).

The prices for final products relating to the cattle sector (beef, veal, milk and milk products) are mainly policy driven by the development of administrative prices. Effects on demand inside of the EU are rather low for these saturated markets. However, the outcomes of the simulation runs depend on the endogenous prices for young animals. For further analysis of the cattle sector, two improvements are envisaged: a split-up of calves into male and female ones and endogenous final weights for fattening processes.

Table 4.5: Animal Production in Europe, Physical Production (1000 tons)

	Base year 1994	Reference 2005	Agenda 2005	Reference to Base year	Agenda to Reference
Meat	24268	24955	24753	2.8%	-0.8%
Beef	7694	7558	7471	-1.8%	-1.2%
Veal	849	775	824	-8.7%	6.3%
Sheep- and goatmeat	1284	1217	1220	-5.3%	0.3%
Pigmeat	16573	17397	17282	5.0%	-0.7%
Poultry	7757	7268	7246	-6.3%	-0.3%
Eggs	4893	5583	5579	14.1%	-0.1%
Milk (unprocessed)	129475	130436	135698	0.7%	4.0%
Cow milk	119741	121347	126497	1.3%	4.2%
Sheep and goats milk	9734	9089	9201	-6.6%	1.2%

Source: CAPRI database and own calculations.

Table 4.5 summarises the results for the two scenarios. Changes inside the cattle sector in Agenda 2000 mainly result from a milk quota increase driving up the dairy cow herd (+4.6%) and a larger suckler cow herd (+2.8%) due to the premium raise. The higher availability of calves compared to the reference run favours the fattening activities. For other meat products, reactions mostly depend on the feed back from the market. The somewhat astonishing substitution between poultry and pig meat in the reference run is based on tariff reductions for poultry meat which leads to an exogenous price shift. Here, further insight in the application of trade policies in the meat markets is clearly necessary. It should be mentioned that tariff impacts on meat markets are generally a sensible and complicated field as instruments relate to specific cuts and qualities whereas the model deals with the combined effect on the raw product price. Presented results clearly reflect the current weighting scheme and must be carefully discussed and eventually re-designed by market experts. Additionally, pig and poultry markets are strongly influenced by assumed market developments in rest-of-the-world as well. Overall, the results show that the system is operational, but underlines the necessity for co-operation with market experts in order to better define trade policy measurements.

4.2.5 Conclusions of the Explorative CAPRI Application

The CAPRI project has been successful in developing a regionalised agricultural information system for the EU. It is now in the position to establish an enduring usefulness for EU- and national policy makers to address the manifold expressed interest during the development phase. In order to insure a survival of the system, a regular update of the database, partial methodological improvements as well as a systematic validation of the model are necessary. It is quite clear that this can only be achieved (1) in the network approach which ensures the in-depth knowledge of regional aspects of agricultural production and the access to national data sources

and (2) in a close dialogue with policy makers to efficiently use the system for policy design and evaluation.

4.3 PMP with Multiple Data Points: A Cross-Sectional Estimation Procedure

4.3.1 Motivation and Overview

CAPRI's concept of the underlying comparative static modelling system combines a supply component comprising about 200 regional programming models with a multi-commodity market model in an iterative fashion to endogenously determine regional supply and national demand quantities, net trade at Member State and EU-level, and equilibrium market prices. The basic question that initiated the research presented in this section is how to specify the regional programming models such that they offer an empirically valid supply response for a large number of crop activities (up to 20).[42]

Aggregate programming models are still widely used for policy relevant analysis of agricultural supply behaviour. Their ability to easily incorporate important policy measures such as quotas and per hectare premiums at a highly differentiated product level, the implied consistency with primary factor constraints during simulations, and the possibility to use explicit assumptions on technology renders this methodological choice preferable to the use of duality based econometric models for many analysts. However, these advantages come at the price of enormous data requirements - which often exclude the compilation of time series - and a typical lack of empirical validation. The CAPRI database offers average yields, average use of variable inputs by production activities, and activity levels at least for the years 1990 to 1995 based on the REGIO database of Eurostat and complementary national statistics.[43] It currently lacks, however, regional stocks on labour and capital and their activity differentiated use as well as a representation of the heterogeneous soil qualities in the EU regions. Consequently, the specification of the regional production technology is not sufficient to avoid overspecialisation of model solutions and to guarantee plausible simulation behaviour based on a typical linear programming formulation. The use of - at the aggregate level - weakly justified rotational constraints or direct bounds on activity levels to better match observed land allocation cannot be seriously considered for policy simulation exercises.

[42] Note that this section repeats a few arguments from the earlier chapters, but within the CAPRI context.
[43] The CAPRI database is currently updated until the year 2000.

PMP (HOWITT, 1995a) promises a remedy: it allows to calibrate insufficiently specified programming models to observed behaviour in an elegant fashion without restricting the model's simulation behaviour by unjustified bounds. Consequently, the application of PMP in policy relevant agricultural supply models - which started already in the eighties (for example HOWITT and GARDNER 1986; HOUSE 1987, KASNAKOGLU and BAUER 1988) - has significantly increased during the last ten years (for example: HORNER et al. 1992; SCHMITZ 1994; ARFINI and PARIS 1995; BARKAOUI and BUTAULT 1999; CYPRIS 2000; GRAINDORGE et al. 2001, HELMING et al. 2001).

However, many modellers have not reflected the arbitrary and potentially implausible response behaviour of the resulting models implied by standard applications of the approach (HECKELEI 1997). PARIS and HOWITT (1998) interpret PMP as the estimation of a non-linear cost function and generalise the specification by employing a 'Maximum Entropy' (ME) procedure. Here we present an approach, which overcomes some of the drawbacks involved in their analysis providing a useful tool for calibration - but more importantly - for the specification of a plausible crop allocation response of aggregate programming models based on observed behaviour.

First we present the idea and associated problems of the specific PMP-approach introduced by PARIS and HOWITT. Then an ME-PMP approach for crop production is designed to exploit information contained in a cross sectional sample to specify - regionally specific - quadratic cost functions with cross effects for crop activities. The approach is applied to CAPRI's regional programming models in France, estimated elasticities are compared to a comparable econometric study, and the programming models are validated in an ex-post simulation exercise.

4.3.2 The Maximum Entropy Approach to PMP

Reminder on PMP

First we remind the reader again of the two phases involved in PMP to calibrate typical linear programming models to observed activity levels (see also chapter 2). The general idea of PMP is to use information contained in dual variables of a linear programming (LP) problem[44] bounded to observed activity levels by calibration constraints (Phase 1), in order to specify a non-linear objective function such that observed activity levels are reproduced by the optimal solution of the new programming problem without bounds (Phase 2).

[44] The method can be applied to non-linear programming problems as well. In order to ease the understanding, a simple but general layout of a LP model is discussed here.

Using a simplified LP formulation designed to determine the profit maximising crop mix, *phase 1* of this procedure is formally described in the following way:

$$
\begin{aligned}
&\max_{\mathbf{x}} \; Z = \mathbf{p}'\mathbf{y} - \mathbf{c}'\mathbf{x} \\
&\text{subject to} \\
&\mathbf{A}\begin{bmatrix}\mathbf{x}\\ \mathbf{y}\end{bmatrix} \le \mathbf{b} \;\; [\boldsymbol{\lambda}] \\
&\mathbf{x} \le \left(\mathbf{x}^{o} + \boldsymbol{\varepsilon}\right) \;\; [\boldsymbol{\rho}] \\
&\mathbf{x} \ge \mathbf{0}
\end{aligned}
\qquad (4.1)
$$

where Z denotes the objective function value, $\mathbf{c}$ and $\mathbf{x}$ are (N×1) vectors of variable cost per unit of activity and production activity levels, respectively, $\mathbf{p}$ and $\mathbf{y}$ are (N×1) vectors of (expected) output prices and sales activity levels, respectively, $\mathbf{A}$ represents a (M×N) matrix of coefficients in resource/policy constraints, $\mathbf{b}$ and $\boldsymbol{\lambda}$ are (M×1) vectors of available resource quantities and their dual variables, respectively, $\boldsymbol{\rho}$ are dual variables associated with the calibration constraints[45], $\mathbf{x}^{o}$ is a (n×1) vector of *observed* production activity levels and $\boldsymbol{\varepsilon}$ denotes a vector of small positive numbers.

The addition of the calibration constraints forces the optimal solution of the LP model (4.1) to almost perfectly reproduce the observed base year activity levels $\mathbf{x}^{o}$, given that the specified resource constraints allow for this solution (which they should if the data are consistent). 'Almost perfectly' is defined by the range of the positive perturbations of the calibration constraints, $\boldsymbol{\varepsilon}$, which are introduced to prevent linear dependencies between resource and calibration constraints. The latter would provoke degenerate dual solutions with marginal values arbitrarily distributed across resource and calibration constraints.

In *phase 2* of the procedure, the vector $\boldsymbol{\rho}$ is employed to specify a non-linear objective function such that the marginal cost of the preferable activities are equal to their respective revenues at the base year activity levels $\mathbf{x}^{o}$. Given that the implied variable cost function has the right curvature properties (convex in activity levels) the solution to the resulting programming problem is equal to the results of (4.1) with respect to activity levels and dual values on the resource constraints, $\boldsymbol{\lambda}$.

[45] The calibration constraints are expressed as *upper* bounds on activity levels. This is sufficient as long as the realisation of the activity provides a positive contribution to the objective function. This should be the case for *expected* profits if positive activity levels are observed. When using realised yields and prices of a calibration year, however, negative profits per activity may occur so that calibration constraints must be formulated as lower bounds as well.

For reasons of computational simplicity and lacking strong arguments for other types of functions, we will illustrate the specification of the parameters in the objective function with the following general version of a quadratic variable cost function[46]:

$$C^v = \mathbf{d}'\mathbf{x} + \frac{1}{2}\mathbf{x}'\mathbf{Q}\mathbf{x} \tag{4.2}$$

where C^v denotes variable costs, $\mathbf{d}$ is a (N×1) vector of parameters associated with the linear term, and $\mathbf{Q}$ is a (N×N) symmetric positive (semi-)definite matrix of parameters associated with the quadratic term of C^v.

The parameters of (4.2) need to be specified such that

$$\frac{\partial C^v(\mathbf{x}^o)}{\partial \mathbf{x}} = \mathbf{M}\mathbf{C}^v = \mathbf{d} + \mathbf{Q}\mathbf{x}^o = \mathbf{c} + \rho\,. \tag{4.3}$$

This specification problem is 'ill-posed', because the number of parameters to be specified (N+N(N+1)/2) is greater than the number of observations (N observations on marginal cost). Traditional econometric approaches could handle this type of problem if an appropriate number of a-priori restrictions on the parameters leave enough degrees of freedom. Most applications of PMP go without any type of estimation by setting all off-diagonal elements of **Q** to zero and calculating the remaining parameters by some standard approach (see section 2.2 for a discussion). Although these approaches work perfectly well with respect to the calibration property of PMP by setting appropriate first order derivatives of the objective function according to (4.3), the resulting simulation behaviour is completely arbitrary (and potentially unsatisfactory, see CYPRIS 2000 and the simulation results below). This is because the response behaviour of the calibrated model depends to a large extent on the *second order derivatives* of the objective function, i.e. on the *change* in marginal cost when activity levels are changing. However, just one observation on dual values of the calibration constraints does not provide any information on this.

Maximum Entropy Specification of the Cost Function

PARIS and HOWITT (1998) suggest to use Maximum Entropy (ME) estimation which allows for a more objective specification of the parameters of the non-linear cost function based on an 'econometric type' criterion. Moreover, it has the potential of incorporating more than one

[46] PARIS and HOWITT (1998) show the general applicability of their approach also with respect to other functional forms. Compared to equation (4.1) they choose, however, a somewhat restricted quadratic functional form by

observation on activity levels into the specification of the parameters and decreases the need to decide on exact a priori restrictions on the parameters. The application of ME to the calibration of programming models comes at a time of significantly increased general interest in entropy techniques by agricultural economists after the comprehensive introduction by GOLAN et al. (1996). Their framework based on probability supports of parameters and error terms allowed to apply the entropy criterion to ill-posed problems in econometrics. Studies in the realm of production economics often focus on the estimation of input allocation to products and estimation of production technologies (for example LENCE and MILLER 1998a and b; LÉON et.al. 1999, ZHANG and FAN 2001). Applications to dual behavioural models are, so far, less frequently observed (OUDE LANSINK 1999b). Note that this section should rather be seen in the context of the PMP literature and consequently does not focus on contributions to the application of entropy techniques in general. However, below we draw upon various of the already mentioned publications when specifying the calibration approach.

To make ME-estimation of the variable cost function (4.2) operational[47] we first need to define support points for the parameter vector **d** and the matrix **Q**. One could centre the linear parameters **d** around the observed accounting cost per unit of the activity, **c**. For example, we could choose 4 support points for each parameter by setting[48]

$$\mathbf{zd}_i = \begin{bmatrix} -2 \cdot c_i \\ 0 \cdot c_i \\ +2 \cdot c_i \\ +4 \cdot c_i \end{bmatrix} \quad \forall i \tag{4.4}$$

In the case of the **Q**-matrix we have to distinguish the diagonal (= change in marginal cost of activity i with respect to the level of activity i) from the off-diagonal elements (= change in marginal cost of activity i with respect to the level of activity j). Given that the *a-priori* expectation for the linear parameter vector **d** are the accounting costs (supports centred around c_i in equation (4.4)), it is consistent with condition (4.3) to centre the support points for q_{ii} around

excluding linear parameters.

[47] see PARIS and HOWITT (1998) for further details and a more extensive motivation of the approach.

[48] The variance of the maximum entropy estimates is negatively correlated with the number of support points defined and has a limit value for an infinite number of support points (see GOLAN et al. 1996, p.139). There is no general rule for the 'right' number of support points, but tests with our models have shown that choosing more than 4 support points does not change the numerical results of the calculated parameter estimates by an extent of any practical relevance.

ρ_i / x_i^o and the off diagonal elements q_{ij} around zero. The centre of the support points ρ_i / x_i^o for the diagonal elements are nonnegative, a necessary condition for convexity of C^v.

A suitable specification for the support points of **Q** would then be

$$(4.5) \qquad \mathbf{zq}_{ii} = \begin{bmatrix} 0 \cdot \rho_i / x_i^o \\ \tfrac{2}{3} \cdot \rho_i / x_i^o \\ \tfrac{4}{3} \cdot \rho_i / x_i^o \\ 2 \cdot \rho_i / x_i^o \end{bmatrix} \quad \forall \ i \ \text{and} \ \mathbf{zq}_{ij} = \begin{bmatrix} -3 \cdot \rho_i / x_j^o \\ -1 \cdot \rho_i / x_j^o \\ +1 \cdot \rho_i / x_j^o \\ +3 \cdot \rho_i / x_j^o \end{bmatrix} \quad \forall \ i \neq j$$

Denoting the probabilities for the K support points $\mathbf{zd}_i$, i = 1,...,N, and $\mathbf{zq}_{ij}$, i,j = 1,...,N, as pd_{ki} and pq_{kij}, respectively, the estimated values of the corresponding parameters are calculated as

$$(4.6) \qquad \begin{aligned} d_i &= \sum_{k=1}^{K} pd_{ki} zd_{ki}, \quad \forall \ i \\ q_{ij} &= \sum_{k=1}^{K} pq_{kij} zq_{kij}, \quad \forall \ i,j \end{aligned} \quad .$$

The ME formulation of estimating the parameters then looks like the following:

$$(4.7) \qquad \begin{aligned} \max_{pd_{ki}, pq_{kij}, d_i, q_{ij}} \quad & H(p) = -\sum_{k=1}^{K}\sum_{i=1}^{N} pd_{ki} \ln pd_{ki} - \sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{N} pq_{kij} \ln pq_{kij} \\ & \text{subject to} \\ & d_i + \sum_{j=1}^{n} q_{ij} \ x_j^o = c_i + \rho_i, \quad \forall \ i \\ & d_i = \sum_{k=1}^{K} pd_{ki} zd_{ki}, \quad \forall \ i \\ & q_{ij} = \sum_{k=1}^{K} pq_{kij} zq_{kij}, \quad \forall \ i,j \\ & \sum_{k=1}^{K} pd_{ki} = 1, \quad \forall \ i \\ & \sum_{k=1}^{K} pq_{kij} = 1, \quad \forall \ i,j \\ & q_{ij} = q_{ji} \quad \forall \ i,j \end{aligned}$$

The entropy criterion in the objective function of (4.7) looks for the set of probabilities which adds the least amount of information - i.e. deviates the least from a uniform distribution over the support points - but satisfies the explicitly shown 'data constraint' of the estimation problem being the marginal cost condition (4.3).

At this point we need to hold for a moment and need to address the question under what conditions a ME formulation for estimating the parameters of the quadratic cost function deems useful. If we have only a 1×N vector of marginal cost available (from calibrating one linear programming problem to one base year solution), the outcome of the estimation and hence the simulation behaviour of the resulting model will be heavily dominated by the supports. Such an application of ME should hence be interpreted as calibrating a cost function based on prior expectations on the parameter values to observed values according to condition (4.3). The entropy criterion works here as a penalty function for the deviation from the prior expectations (centre of supports) and the term estimation for the calibration process may be misleading. The approach with just one observation on marginal cost could be sensibly applied to derive a cost function based on specific prior information, for example on supply elasticities[49] or to exogenously given yield functions (HOWITT 1995a).

The example defined according to equations (4.4) - (4.7) is however not a meaningful application for such a calibration process as supports were defined without any valuable prior information on the cost function. Specifically, the ME problem will reach its optimum when the probabilities follow a uniform distribution, since the centres of the support ranges already satisfy the data constraints. The resulting parameter estimates will be exactly the ones implied by the 'standard approach' as defined in HECKELEI (1997) or section 2.2, i.e. linear parameters of the cost function are equal to the respective activity's accounting costs c_i, the off-diagonal elements of the **Q**-matrix are zero, and the diagonal elements are equal to ρ_i / x_i^o. The simulation behaviour of the resulting model is arbitrary as it completely depends on the arbitrary specification of the support values.

For similar reasons, the approach of PARIS and HOWITT (1998) who reparameterise the **Q**-matrix based on a **LDL**' (Cholesky) decomposition to ensure appropriate curvature properties of the estimated cost function should - in our view – only be seen as a demonstration on how to combine ME and PMP. The choice of their support values is not based on prior information. They centre the elements of **D** around 1.33 ρ_i / x_i^o and the supports of **L** around zero. Together with the complex (and even order-dependent) relationship between the matrices **L**, **D** and **Q**, this implies rather non-transparent a-priori expectations for the parameters of **Q**. The nonzero cross costs effects of activities obtained from their ME solution is merely based on this technically motivated choice of support points.

[49] See sections 2.2 and 3.3.4, the latter in the context of calibration without PMP.

In contrast to the examples given above, we now suggest an approach based on a cross-sectional sample of marginal cost from a set of regional programming models. We apply the term 'estimation' for this procedure, since several regional vectors of marginal costs are used to specify the cost functions. The choice of ME instead of other estimators is motivated by the fact that we have still negative degrees of freedom. The discussion will mostly concentrate on necessary parametric restrictions across regions to accommodate for regions with different sizes and crop rotations. Additionally, we provide a solution to the curvature problem which allows the definition of support points for the actual parameters to be estimated by incorporating a **LL'** decomposition as direct constraints of the estimation problem.

4.3.3 A PMP-ME Approach Based on a Cross Sectional Sample

The first part of this section presents a rationale for the approach and introduces the most important parts of the mathematical formulation. The second part delivers some details on an application for CAPRI's regional programming models for France and presents results of elasticity estimates and an ex-post validation for the simulation behaviour of the specified model across the CAP-reform of 1992.

Rationale

Our objective here is to estimate a quadratic cost function with cross cost effects (full **Q**-matrix) between *crop production activities*. Suppose one can generate *R* 1×N vectors of marginal costs from a set of *R* regional programming models by applying the first phase of PMP. In our example, N represents 18 crops and *R* the 22 French NUTS II regions. In order to exploit this information for the specification of quadratic cost functions for all regions, we need to define appropriate restrictions on the parameters across regions, since otherwise no informational gain is achieved.

Consider the following suggestion for a regional vector of marginal cost:

$$\begin{aligned} &\mathbf{MC}_r^v = \mathbf{d}_r + \mathbf{Q}_r \mathbf{x}_r \quad \forall r \\ &\mathbf{Q}_r = \left(\mathrm{cpi}_r\right)^g \ \mathbf{S}_r \mathbf{B} \mathbf{S}_r' \quad \forall r, \text{ with } s_{rii} = \sqrt{1/x_{ri}^o} \end{aligned} \tag{4.8}$$

where $\mathbf{d}_r$ is a (N×1) vector of linear cost function parameters in region r, $\mathbf{Q}_r$ represents a (N×N) matrix of quadratic cost term parameters in region r, cpi_r stands for regional 'crop profitability index' defined as the relation between the regional and average revenue per hectare

$(\mathbf{p}'\mathbf{y}_r / L_r) / \left(\sum_r \mathbf{p}'\mathbf{y}_r / \sum_r L_r \right)$ where L_r is land available, g is a parameter determining the influence of the crop profitability index, $\mathbf{S}_r$ constitute (N×N) diagonal scaling matrices for each region r, and finally **B** is a (N×N) parameter matrix related to $\mathbf{Q}_r$.

The rationale for (4.8) can best be inferred from a didactic example shown in Table 4.6, based on a linear programming model with 2 crops, 2 regions and a land constraint. Rows 1-3 present the observed base year data – total revenues, accounting costs and activity levels - from which the dual values of the calibration constraints, marginal costs (rows 4-6) as well as average revenues and the crop profitability index (row 7-8) can be deducted.

Table 4.6: 2-region/2-Crop Example for Cost Function Specification

	Items	Symbols in Text	Unit	Region 1		Region 2		National average	
				Cereals	Other Crops	Cereals	Other Crops	Cereals	Other Crops
1	Revenue	**p'y**	EURO/ha	1000.00	900.00	800.00	700.00	995.12	842.86
2	Accounting cost	**C**	EURO/ha	550.00	500.00	450.00	400.00	547.56	471.43
3	Activity level	**X**	Ha	40.00	10.00	1.00	4.00	41.00	14.00
4	Dual on calibration constraint	ρ	EURO/ha	50.00	0.00	50.00	0.00		
5	Dual on land constraint	λ	EURO/ha	400.00		300.00			
6	Marginal Costs	**Mc**	EURO/ha	600.00	500.00	500.00	400.00	597.56	471.43
7	Average Revenue		EURO/ha	980.00		720.00		956.36	
8	Crop profitabilty index	Cpi_r		1.02		0.75		1.00	
9	Average Marginal cost	**amc**						597.56	534.49
								534.49	471.43
10	Scaled supports for uniform **B**	**zbs**						5.00	0.00
								0.00	5.00
11	A priori expectation for uniform **B**	E[**zb**] = E[**zbs**]*__amc__						2987.80	0.00
								0.00	2357.14
12	Elements of scaling matrix **S**	$S_{ii} = (1/x_i)^{0.5}$		0.16	0.05	1.00	0.50	0.16	0.04
				0.05	0.32	0.50	0.50	0.04	0.27
13	Exponent of crop profitability index	g						1.00	
14	Influence of crop profitability index	Cpi^g		1.02		0.75		1.00	
15	A priori expectation $\mathbf{Q}_r$	$E[\mathbf{Q}_r] = cpi^g$*__S__*E[__zb__]*__S__'		76.54	0.00	2249.37	0.00	72.87	0.00
				0.00	241.54	0.00	443.64	0.00	168.37
	Percentage change of marginal costs for a 1% increase in levels								
16	using **B** only			199.19	47.14	5.98	23.57	205.00	70.00
17	using **SBS'**			4.98	4.71	5.98	5.89	5.00	5.00
18	using $\mathbf{Q}_r$			5.10	4.83	4.50	4.44	5.00	5.00

Contrary to the ultimate application, the matrix **B** as shown in the last columns of row 11 is given, not estimated. It is defined such that the relative increase of marginal costs for a 1% increase in levels is equal to 5% at national level. In order to motivate the scaling with $\mathbf{S}_r$, we have a look at the implied elasticities of marginal costs to changes in activity levels as shown in row 16 if the scaling vectors **S** are left out. In that case, elasticities are a direct function of observed activity levels: the smaller the level, the smaller the elasticity. Including the scaling vectors, as shown in row 17, provides a more plausible parameter restriction.

The term $(cpi_r)^g$ which reflects *differences in regional profitability* is supposed to capture the economic effect of differences in soil, climatic conditions etc. The magnitude of the effect on the marginal cost function estimated by the exponent g. A negative g, for example, would imply that specialising in a certain crop is penalised less in a region with cropping conditions above average since, ceteris paribus, $\mathbf{Q}_r$ is smaller than average in this case.

The specification implies that - apart from the effect of the crop profitability index - the $\mathbf{Q}_r$'s are identical for regions with the same crop rotation. We motivated the use of more than one observation by the fact that second order derivatives of the cost function strongly influence the simulation behaviour of the model. Where does this information hide in equation (4.8)? Observed rotations and marginal costs recovered by the calibration step differ between regions. The matrix **B** - common across regions - is estimated as to describe the differences in marginal costs depending on the differences in levels. The parameters are now estimated such that changing region i's rotation to the rotation in region j causes changes in marginal cost matching the observed differences between the two regions (again apart from the effect of the crop profitability index). This is the important contribution of the cross-sectional analysis: the simulation behaviour resulting from the ME problems is not longer depending in an arbitrary way on the support points, but is based on a clear hypothesis about the relation between crop rotation and marginal costs.

The general formulation of the corresponding ME problem is now straightforward:

$$
\begin{aligned}
\max_{pd_{kir},pb_{kij},pg_k,d_{ir},b_{ij},g} \quad & H(p) = -\sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{r=1}^{R} pd_{kir} \ln pd_{kir} - \sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=i}^{N} pb_{kij} \ln pb_{kij} - \sum_{k=1}^{K} pg_k \ln pg_k \\
& \text{subject to} \\
& d_{ir} + cpi_r^g \cdot \sum_{j=1}^{N} s_{ii} s_{jj} b_{ij} x_{jr}^o = c_{ir} + \lambda_{ir}, \quad \forall\, i, r \\
& d_{ir} = \sum_{k=1}^{K} pd_{kir} zd_{kir}, \quad \forall\, i, r \\
& b_{ij} = \sum_{k=1}^{K} pb_{kij} zb_{kij}, \quad \forall\, i, j \\
& g = \sum_{k=1}^{K} pg_k zg_k \\
& \sum_{k=1}^{K} pd_{kir} = 1, \quad \forall\, i, r \\
& \sum_{k=1}^{K} pb_{kij} = 1, \quad \forall\, i, j \\
& \sum_{k=1}^{K} pg_k = 1 \\
& b_{ij} = b_{ji}, \quad \forall\, i, j
\end{aligned}
\tag{4.9}
$$

The current formulation in (4.9) does not guarantee that a *positive (semi-)definite* matrix **B** - and consequently - positive (semi-)definite matrices $\mathbf{Q}_r$ will be recovered. A violated curvature property might result in a specification of the objective function that does not calibrate to the base year, since only first order but not second order conditions for a maximum are satisfied at the observed activity levels. In order to circumvent the problems with the **LDL'** reparameterisation of PARIS and HOWITT described above, a 'classic' Cholesky decomposition of the form **B** = **LL'** is used *indirectly* as additional constraints of the ME problem (4.9) in the form of[50]

$$
\begin{aligned}
& l_{ii} = \sqrt{E\left[b_{ii}\right] - \sum_{k=1}^{i-1} l_{ik}^2} \quad \forall \;\; i, j \\
& l_{ij} = \left(E\left[b_{ij}\right] - \sum_{k=1}^{i-1} l_{ik} l_{jk} \right) \Big/ l_{ii} \quad \forall \;\; i, j \;\text{ and }\; j > i
\end{aligned}
\tag{4.10}
$$

Because **B** is supposed to be a symmetric and positive (semi-)definite matrix, the $l_{i,i}$ must always be positive and real (GOLUB and VAN LOAN 1996). Appropriate lower bounds on $l_{i,i}$ deviating

[50] The two different forms of the Cholesky decompositions are related in the following way: Replacing the 'ones' on the diagonal of the lower triangular matrix **L** of **Q** = **LDL'** with the square roots of the corresponding diagonal elements of **D** allows to write **Q** = **LL'**.

from zero avoid zero divisions during estimation. Due to the properties of positive (semi-) definite matrices, the regional matrices $\mathbf{Q}_r$ calculated according to the second equation in (4.8) are positive (semi-)definite if **B** exhibits this property. A separate enforcement of curvature for each $\mathbf{Q}_r$ would be computationally infeasible which potentially restricts the type of alternative parameter restrictions across regions if this curvature solution is employed.[51]

An Application to Crop Production in France

In this section we describe an application and ex-post validation of the suggested approach for the regional programming models of the CAPRI system for France. Before turning to the results the specification of the support points for the parameters is presented:

The support points for the exponent g of the crop profitability index cpi_r in (4.9) are defined as

$$\mathbf{zg} = \{-2, -\tfrac{2}{3}, +\tfrac{2}{3}, +2\} \tag{4.11}$$

so that the influence of the crop profitability index covers the range from $1/cpi_r^2$ to cpi_r^2 and the support of g is centred around 0. The estimation came out with a slightly negative value which implies that cropping conditions above average allow crop specialisation with marginal cost increases below average.

The crop and region specific linear terms **d** reflect marginal costs when all production activity levels **x** are zero. Since an interpretation in economic terms is hardly possible and irrelevant - especially as 'fallow land' is one of the production activities - the spread of the support points **zd** is consequently set to a very wide interval around the observed costs. The spread 180 times the national average in revenue per ha.[52]

$$\mathbf{zd} = \mathbf{c}_r + \{-90, -30,\ 30,\ 90\}\ \sum_r \mathbf{p}'\mathbf{y}_r \Big/ \sum_r L_r\ . \tag{4.12}$$

Let $\overline{MC}_i$ be the land-weighted average of marginal cost for crop i across regions. The support points for **B** are then defined as follows (see rows 9-11 in Table 4.6 as well):

[51] In earlier tests, a pragmatic solution was chosen for the curvature problem by forcing the first and second order minors of **B** to have the appropriate sign and restricting all off-diagonal elements to be smaller than diagonal elements during the ME step. The resulting matrix was then - if necessary - treated by a so-called 'modified' Cholesky-decomposition which ensures definiteness by employing optimal correction factors to the diagonal elements (GILL et al. 1989: 108 ff.). This procedure has proven to be operational for very large matrices.

[52] With this support point formulation, the linear terms $\mathbf{d}_r$ could also be viewed as the sum of a predetermined parameter vector $\mathbf{c}_r$ and a crop and region specific error term which is centred around zero. Consequently, the specification is numerically equivalent to a generalised ME formulation with error terms. We opted for the representation above, because the 'error term' is ultimately kept in the specification of the objective function so that the resulting programming models calibrate exactly to observed activity levels.

$$\mathbf{zb}_{ij} = \mathbf{zbs}_{ij} \quad \mathbf{amc}_{ij}$$

$$\text{where} \quad \mathbf{zbs}_{ij} = \begin{bmatrix} \{0.001,\ 3.3,\ 6.66,\ 10\} & \forall & i = j \\ \left\{-2,\ -\tfrac{2}{3},\ \tfrac{2}{3},\ 2\right\} & \forall & i \neq j \end{bmatrix} \quad \text{and} \quad \mathbf{amc}_{ij} = \tfrac{1}{2}\left(\overline{MC}_i + \overline{MC}_j\right) \tag{4.13}$$

According to the spread defined by **zbs**, the supports **zb** for **B** are defined such that changing the activity level of crop i by 1% increases marginal cost of this crop between zero and ten percent at the mean of the observations. The cross effects are symmetrically centred around zero and allow for a change between -2% and +2% of the average marginal costs of crop i and j, **amc**. This support point definition clearly introduces prior information. The elements of **B** will be drawn towards the centre of the support intervals by the entropy criterion as much as the data constraints allow. In addition we excluded (the theoretically impossible) negative values for the diagonal elements and restricted the cross effects to be small relative to the own activity level effects on marginal cost. Nevertheless, the spread of the support points specification leaves considerable freedom for obtaining a wide range of implied elasticities.

The determination of support points in the context of ME and GME (Generalised ME which includes error terms in data constraints) is a delicate problem and therefore deserves some further discussion: There seems to be a great desire to determine support points objectively and to avoid prior information as much as possible. LÉON et. al. (1999), for example, employ the normalised entropy measure to judge the 'superiority' of different (predefined) symmetric and asymmetric support point specifications.[53] This measure reaches its maximum when the estimated parameters do not deviate at all from the a-priori expectations defined by the support values.[54] Consequently, it allows to compare different support point specifications with respect to their compatibility with the data constraints. The measure does not allow, however, to identify an optimal set of support values for an underdetermined estimation problem. Just as there is an infinite number of parameter vectors satisfying the data constraints, there is as well an infinite number of support definitions with prior expectation equal to these parameter vectors. All these support point specifications obtain the same value of the normalised entropy measure, but not the same parameter estimates. Therefore, we did not consider this measure for the choice of support values here.

[53] See also GOLAN et al. (1996) for a discussion of 'Normalised Entropy' and its use in various applications.

[54] Generally, prior expectations are defined as a weighted average of support values. In the ME case, the weights are probabilities following a uniform distribution. In the CE case, the weights are the probabilities as defined by the reference distribution.

Other research focussing on the idea of using purely 'data-based' supports (VAN AKKEREN et al. 2001) show advantages over classical estimation techniques in some ill-conditioned (e.g. multicollinear) data situations but well posed with respect to the number of observations. Those techniques obviously cannot make up for limited data information. From our point of view it should simply be accepted that a small number of observations relative to the number of parameters imply little information and that ME and GME succeed in these situations, only because they allow to flexibly incorporate prior information by restricting the parameter space. There is certainly the danger of introducing a strong bias if the prior is formulated very tight and far off the true value. In the context of GME estimation of the linear model, however, it can be taken as some comfort that the estimator is consistent under general regularity conditions as long as the true value of the parameters is within the support range (see MITTELHAMMER and CARDELL 2000).

Above, we tried to make our a-priori information as transparent as possible and chose to use a uniform distribution where the centres are the prior expectations. Note that this is numerically equivalent to a cross entropy (CE) approach with this uniform distribution serving as the reference distribution. Other possibilities to represent prior information include differentiated prior weights in the CE reference distribution or asymmetric support point spacing in ME and CE contexts. These methods provide flexibility in expressing just the prior information that is available, but – to our knowledge – there is no objective criterion that makes one approach generally superior to the others. At some point, there might be measures to compare the penalty involved for deviating from the prior information for the different approaches and this will improve transparency (see PRECKEL 2001 for looking at the entropy criterion from a penalty view).

Returning from this general support point discussion to our specific case, we repeat that the specifications in (4.12) and (4.13) imply some prior information, but the support spread leaves considerable ranges for the parameters. Also the influence of the support points on the estimation outcomes becomes considerably smaller with an increasing number of observations and allowing for this to happen is a major objective of our approach in contrast to previous PMP applications.

The approach discussed in the previous section estimates a non-linear cost function depending on crop production activity levels based on observed regional differences in marginal costs at just one point in time. Naturally, doubt may be raised if that cross-sectional information can be just mapped in the time domain by assuming that changes in crop rotation over time in each single region have a similar effect on variable costs as the differences in observed crop rotations for a

set of regions at one point of time. We consequently check below the resulting simulation behaviour of the models in an ex-post simulation exercise.

We took three year averages both for the calibration and simulation year based on data in the CAPRI data base for the 22 NUTS-2 regions in France.[55] Given data availability, we used years 1989 to 1991 ('1990') for the calibration and 1993 to 1995 ('1994') for the simulation. The move from 1991 to 1994 has the advantage that the 1992 CAP reform lays just in between which offers a good opportunity to test the model under a significant policy change. However, some restrictions apply: We had no data on the participation in voluntary set-aside programs before the CAP-reform - therefore important information was left out in the calibration step. Naturally, no data on obligatory set-aside and non-food production, both introduced by the 1992 CAP reform, entered the calibration for 1990. We therefore had to make some assumptions regarding these activities:

- The parameters in **d** and **B** relating to *voluntary and obligatory set-aside* were set equal to the ones obtained for *fallow land* in 1990, assuming that they have the same rotational effects as represented by the cost function. Nevertheless, voluntary and obligatory set-aside are still treated in the simulation according to the policy formulation in the CAP-reform, i.e. they are linked to the production of 'grandes-cultures' in the appropriate way (see below).
- The driving forces of *non-food production on set-aside* were unknown to us with respect to hard quantitative information. Therefore, we fixed non-food production to known levels in 1994. As non-food has a share around 10% on oilseeds in total, the resulting improvement in the model's fit is not dramatic. We also applied this assumption to the other approaches which are compared to our ME-PMP calibrated model.

The set-aside regulation is modelled by constraints: the obligation must be fulfilled by an appropriate level of obligatory set-aside or non-food production on set-aside. Voluntary set-aside may be added as long as the sum of total set-aside including non-food production does not exceed 33% of the endogenously determined 'grandes cultures' area. Premiums are cut if regional base areas are exceeded. As the presented ME-PMP approach is only suitable for annual crops, we fixed animal production and perennials to observed levels in 1994. Apart from the sugar beet quota and the land restriction, no other constraints enter the model specification.

[55] The employment of three year averages is motivated by the stochastic nature of yields and prices in order to avoid projections of single outcomes into the future.

The ME problem (4.9) was successfully solved with the General Algebraic Modelling System (GAMS, BROOKE et al. 1988) using the solver CONOPT2. It should be noted here that a powerful solver for this type of optimisation problem is necessary, especially due to the considerable non-linearity introduced by the Cholesky decomposition constraints (4.10).

We started the evaluation of the results with simulation experiments based on partial, 10% increases of product prices and calculated the aggregated national percentage change in area related to the price change. Table 4.7 shows selected elasticities which are somewhat comparable to the 'classical' econometric estimates provided by GUYOMARD et al. (1996) with respect to product differentiation and scope.

Table 4.7: Price Elasticities of Supply for Selected Crops– National Aggregate France

	Soft Wheat	Maize	Barley	Rapeseed	Sunflower	Soya
Soft Wheat	**1.322** [0.715]	-0.075 [-0.303]	-0.443 [-0.010]	-0.076 [-0.007]	-0.039 [-0.008]	-0.003 [-0.001]
Maize	-0.165 [-0.624]	**0.653** [1.630]	-0.056 [-0.041]	-0.004 [-0.031]	-0.009 [-0.038]	-0.003 [-0.002]
Barley	-1.555 [-0.042]	-0.105 [-0.097]	**2.647** [0.351]	-0.144 [-0.002]	-0.075 [-0.003]	-0.008 [-0.000]
Rapeseed	-0.939 [-0.079]	-0.041 [-0.033]	-0.453 [-0.025]	**1.457** [0.428]	-0.065 [-0.091]	-0.005 [-0.017]
Sunflower	-0.540 [-0.111]	-0.066 [-0.046]	-0.216 [-0.036]	-0.047 [-0.048]	**1.126** [0.223]	-0.006 [-0.024]
Soya	-0.302 [-0.351]	-0.224 [-0.144]	-0.218 [-0.112]	-0.036 [-0.152]	-0.057 [-0.403]	**1.861** [3.701]

Source: Own Calculations and GUYOMARD et al. (1996). Supply in rows and changed prices in columns. Reported elasticities are calculated as average percentage supply change (change in land allocation due to fixed yields) per one percent price change. The simulations are based on a 10% increase in the respective crop prices. Values in brackets are the (rounded) supply elasticity estimates reported in Table 2 of GUYOMARD et al. (1996).

The estimates of *own price* elasticities are on average larger than their econometric counterparts (reported in brackets), but not uniformly so. The own price response of maize and soya is considerably below the values of GUYOMARD et al. (1996) Generally, the estimated own price elasticities are smaller than the typical supply responses implied by LP's or standard PMP-procedures (see for example CYPRIS 2000 and the subsequent simulation exercise). *Cross price* elasticities are also within the general magnitude of the econometric estimates, but they show clearly different structures of substitution between the crops. For example, with an increase of the soft wheat price, barley and rapeseed show the strongest (percentage) reductions in Table 4.7.

Those responses are rather small in the case of GUYOMARD et al. (1996), where maize is the main crop substituted by the increasing wheat production. One should not forget, however, that the theoretical structure of the two underlying models (fixed versus variable input and output coefficients) as well as the employed data base (cross sectional versus time series) differ between the two sets of estimates which limits their comparability.

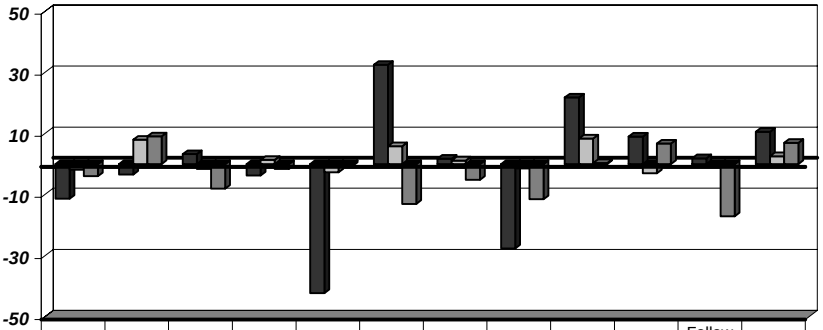
The comparison of the resulting partial supply responses with values of estimated behavioural functions is certainly interesting. However, the assessment of the simulation behaviour across larger economic and policy changes is closer to the ultimate purpose of our model. Therefore, we designed an ex-post simulation experiment as described above, results of which are rarely generated in the context of programming models, but are very informative from our point of view:

In order to judge if the new methodology has comparative advantages, we included a 'standard PMP' approach in the ex-post validation as well. Here, only diagonal elements of **B** are specified such that the linear and quadratic terms for each production activity i implicitly define *average variable cost* matching the observed accounting cost c_i for the base year. In the case of the quadratic cost function this implies that

$$(4.14) \qquad b_{ii} = \frac{2\rho_i}{x_i^o} \text{ and } d_i = c_i - \rho_i \quad \forall i \,.$$

Furthermore, we defined an intelligent 'no-change' forecast by taking 1990 levels of annual crops reducing them - where applicable - by set-aside obligations. The resulting areas were then made consistent to the available land in 1994.

Figure 4.4: Percentage Deviation of Simulated from Observed Production Activity Levels for France

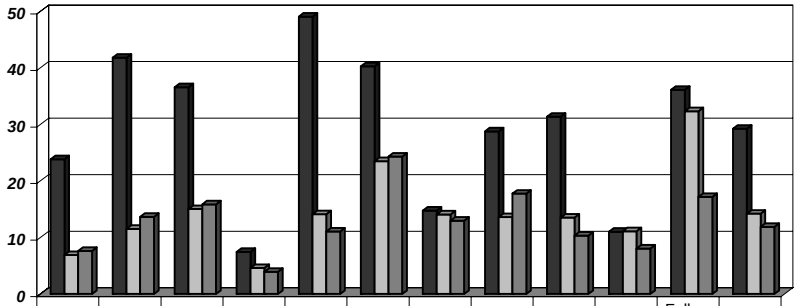


	Wheat	Barley	Maize	Cereals	Sun-flowers	Rape	Oil seeds	Pulses	Pota-toes	Arable Fodder	Fallow Set aside	Total
Standard	-11	-3	3	-4	-42	33	2	-28	22	9	2	11
ME	-2	8	-2	1	-3	6	1	-1	8	-3	0	3
No change	-4	9	-8	-2	-1	-13	-5	-12	0	7	-17	7

Source: Own calculations

Figure 4.4 shows the percentage national deviation of 'simulated' production activity levels from the observed activity levels in 1994 for the three approaches. The 'standard approach' shows rather high deviations for some major crops. Somewhat surprising, the 'no-change' forecast is comparatively close to observed production activity levels. Apparently - the 1992 CAP reform had - at least in France - a relatively small impact on the aggregate crop rotation apart from the set-aside effect. With this in mind, the fit of the ME-PMP approach based on the cross sectional sample is rather promising: apart from sunflowers and potatoes it provides better simulated values than the 'no-change' results. The sum of absolute deviation in levels weighted by the observed levels amounts just up to 3% (see 'Total').

Figure 4.5: Mean Absolute Percentage Deviations of Simulated From Observed Production Activity Levels Across Regions



	Wheat	Barley	Maize	Cereals	Sun-flowers	Rape	Oil seeds	Pulses	Pota-toes	Arable Fodder	Fallow Set Aside	Total
Standard	24	42	37	7	49	40	15	29	31	11	36	29
ME	7	12	15	5	14	23	14	14	14	11	32	14
No change	8	14	16	4	11	24	13	18	10	8	17	12

Source: Own calculations

However, the variation in regional forecasting errors is at least of the same importance. The results in Figure 4.4 could be a rather 'lucky' outweighing of regionally large under- and overestimating of activity levels. Therefore, we additionally checked the regional fit by calculating mean absolute percentage deviations over regions presented for the most important activities and aggregates in Figure 4.5. The standard approach was again no real competitor. However, the performance of the ME-PMP approach is about the same as 'no-change' apart from the aggregate of fallow land and set-aside. As explained above, problems could be expected here as no substantial information entered the calibration step.

4.4 Critical Evaluation of the Estimation Approach and Outlook

At the time of execution, the application of PMP based estimation presented above was the first one with multiple observations on production programs.[56] It provided a new approach addressing the rather arbitrary specification of the non-linear objective function in previous PMP applications. The use of a cross-sectional sample of regional production programs combined with a clear hypothesis on the general relation between marginal cost and crop rotations allowed to base the model specification and the implied supply response on observed differences in behaviour. The resulting programming models fulfil the objective to generate a plausible national and regional supply response within the CAPRI modelling system which was validated ex-post across a major change of European agricultural policy. This ex-post simulation exercise – rarely executed or at least published in the context of aggregate programming models – also demonstrated the superiority of the approach to standard applications of PMP.

Nevertheless, the more general findings on the calibration and estimation of programming models presented in chapter 2 and 3 are largely not reflected by this approach. Therefore, a critical evaluation and an outlook to envisaged and currently explored changes to the specification of regional programming models in CAPRI shall be given here.

The most important point to reflect is that the use of the PMP procedure itself is neither necessary nor recommendable for future specifications. There is no need to employ dual values of the calibration constraints of a linear programming model ρ to specify additional non-linear terms of the objective function. Instead, one can start with the model which is – in the analyst's view – a good representation of regionally aggregate crop supply response and simultaneously constitutes an appropriate simulation tool for the policy program evaluations to be performed in the future. The direct use of the corresponding first order necessary conditions as estimating equations (see sections 2.2.4 and the methodological demonstration in sections 3.3 to 3.5) avoids the fundamental problem of PMP which leads to inconsistent parameter estimates even in the case that the assumed model is a good approximation of the true data generating process (section 2.2.3).

Another shortcoming of the application above is the use of supports for *parameters* of the model. Although still the common procedure in GME approaches with limited data information, it is

[56] PARIS 2001 presents the second PMP-type application with multiple observations known to the author. His approach, however, suffers from the same problems regarding curvature imposition and arbitrary support point

often not a good representation of the type of prior information typically available. Sections 3.3 and 3.4 demonstrated ways to incorporate prior information on *functions of parameters* such as elasticities and shadow prices of resources and showed their general ability to stabilise estimates for small sample sizes. The support point definition for the matrix **B** in (4.13) is certainly chosen to allow estimates of any sensible derivative of marginal cost with respect to land allocation. However, the centres of the supports as the prior expectation cannot be directly translated to the implied supply elasticities, because those also strongly depend on the constraint structure of the programming model.[57]

A major concern of future specifications, however, should be to increase the amount of data information during estimation and potentially render the use of additional prior information unnecessary. The pure cross-sectional data base employed above still leaves considerable doubt on whether the resulting model specification really provides a valid supply response over time, despite the comparatively promising ex-post simulation exercise. It could just be a lucky coincidence that the results of the model were close to observed changes in land allocation. After all, the parameter estimation is based upon the assumption that observed differences of marginal cost between regions translate into marginal cost changes within regions. Time series data give a more direct observation on adjustments to changing economic incentives. Changes in land allocations from year to year in a certain region are truly observed behavioural changes of one economic unit and not just states of different economic units. The CAPRI data base is currently updated until the year 2000. If completed, a pooled cross-sectional data set is available resulting in, for example, 242 observations for France (11 years times 22 regions). This provides a solid base for estimation even if additional region specific parameters are considered.

Preliminary tests estimating regional quadratic cost functions for France with the already available 6 observations in the time domain (132 observations altogether) were performed. The employed GME estimation procedure relies on the general approach presented in sections 3.3 to 3.5. Compared to the application above, this specifically implies that

- the estimation employs first order necessary conditions as estimating equations instead of 'PMP-marginal cost equations'

definition as in PARIS and HOWITT (1998) explained above. It also does not provide a plausible scaling of the cost function for different sizes of incorporated economic units.

[57] See the calibration exercises in section 3.3.4 for the considerable error that is involved if priors on elasticities do not reflect the constraints.

- the statistical model assumes stochastic error terms for each endogenous variable (land allocation) and not for each equation (as implicitly done above by the region- and crop specific constants)
- no supports are defined for parameters.

The increased number of observations combined with the differentiated set of crop activities causes considerable computational demand. Initial numerical difficulties could be overcome, however, so that various versions with and without prior information on supply elasticities were explored. Results generally show supply elasticity estimates that are – on average - somewhat smaller than the ones presented in Table 4.7. It is interesting to note that the prior information did not have a significant impact on the estimation results and will not be employed at least for this model.

This leaves the final question with respect to the model structure itself. So far, simulations with the CAPRI model are based on exogenous input and output coefficients. This has several advantages with respect to scenario definitions and policy implementation. Assumptions on technical progress in reference and simulation run can be made explicit and directly translate into changed input-output combinations and corresponding economic incentives of the different production activities. Specific environmental policy measures with premium payments related to a reduced input use (for example fertiliser and pesticides) are easily and directly representable. Production programs are straightforwardly linked to environmental indicators based on typically available engineering type information from natural scientists. However, all this comes at the cost of the – at least at the aggregate level – erroneous assumption that intensity and productivity per hectare of land do not change with changing prices and policy measures. This might cause significant model errors for simulations across major price and policy changes. It also implies that the quadratic non-linear terms in the objective function do not represent any 'cash cost' but instead risk premiums and a less than satisfactory compensation for aggregation errors or unknown resource constraints.[58] Consequently, estimations of land allocation models based on crop specific profit functions as laid out in sections 2.5.1 and 3.5, or alternatively, a multi-output profit function concept presented in section 2.5.4 are currently explored. The ultimate decision on the structure of the modified CAPRI supply module will not only depend on these estimation results, but also on a careful evaluation of how transparent and suitable the resulting model structure is for policy evaluation and consulting purposes.

4.5 Summary of Chapter 4

This chapter introduced the concept of the large scale policy information system CAPRI and presented results of an explorative application to evaluate agricultural policy changes in the cereal and beef sector implied by Agenda 2000. The modelling system combines EU-wide full coverage of the agricultural sector at a regionally differentiated level. Regional programming models allow to closely implement major policy measures of the CAP on the supply side. The integration with commodity markets at member state, EU and world level including relevant trade policy measures is realised by an iterative market clearing process. CAPRI thereby offers a comprehensive tool for the evaluation of EU agricultural policy programs. In addition, links to environmental indicators carry the analysis beyond the evaluation of effects related to agricultural commodities typically performed with agricultural sector models.

In view of the overall objective of the study, the specification of non-linear cost functions for CAPRI's regional programming models received special attention. The approach can be characterised as an application of PMP to multiple observations. The suggested methodology is designed to exploit information contained in a cross sectional sample to specify - regionally specific - quadratic cost functions with cross effects for crop activities. It also provides a solution to the curvature problem. The approach is applied to regional programming models for 22 regions in France. An ex-post simulation across the 1992 reform of the Common Agricultural Policy (CAP) shows plausible results with respect to the simulation behaviour of the resulting models. Paths for extensions and improvements of this methodology are discussed in light of the findings presented in chapters 2 and 3.

[58] see sections 2.4 and 2.6 for a discussion of these topics.

5 Summary and Conclusions

It was the overall objective of this study to close the methodological gap between the specification of explicit constrained optimisation models and dual systems of output supply/input demand equations for representing multi-output, multi-input agricultural supply behaviour. The pursuit of this goal was motivated by the hope to combine the flexibility of programming models in representing technological constraints and political measures with the statistical specification of model parameters based on observed behaviour. The results can be summarised by the following points:

1) Positive Mathematical Programming (PMP) as the most prominent calibration approach for programming models to date has been reviewed. The introduction of non-linear objective function terms based on dual values of calibration constraints reproduces observed behaviour of a base period and provides a smooth simulation response to changing economic conditions. In these respects the approach is superior in representing aggregate supply response compared to linear programming models used in the past. However, many of the different variants of this approach imply a rather arbitrary quantitative reaction, because information on supply response is missing in the case of just one base year observation on a production program and the parameter choice is based on ad-hoc assumptions to solve the underlying underdetermined specification problem. It was argued that only the introduction of prior information such as exogenous elasticities and/or parameter *estimation* using multiple observations can provide satisfactory solutions.

2) The evaluation of PMP's suitability to provide systems of estimating equations to be used in the context of multiple observations on production programs followed. It could be shown that the structural discrepancy between the 'first-phase-model' with calibration constraints and the resulting model results in data constraints which do not allow to consistently estimate model parameters under the assumption that the final model structure is an appropriate approximation of the true data generating process. Problems arise due to the inconsistent determination of dual values of the constraints which in turn imply a wrong marginal specification of the non-linear objective function terms.

3) A simple but general alternative approach to calibrate or estimate programming models was introduced which uses first order necessary conditions derived from the desired model structure as estimating equations. It allows to simultaneously estimate model parameters and dual values of constraints. This approach is not only suitable for typical PMP models but can

be applied to general programming model specifications as well. It has the potential to provide an equivalent and at the same time more flexible way for estimating constrained optimisation models compared to traditional econometric approaches based on duality theory.

4) In order to address a further shortcoming of typical PMP applications and to derive corresponding first order necessary conditions for estimation, rationalisations of non-linear terms in profit maximisation models with Leontief technology were considered. The inclusion of crop specific land supply functions to represent land heterogeneity was shown to result in a programming model equivalent to PMP models with non-linear costs in activity levels. A more general look at the underlying aggregation problem if the distribution of land qualities is not available lead to a variant of 'convex combination constraints' where a frontier of extreme points is approximated by a non-linear constraint. The same specification can be interpreted as representing an unknown 'operating capacity' constraint.

5) To show the applicability of the estimation principle to model structures underlying previous econometric estimations of behavioural functions, fully rationalised profit maximisation models under general technologies with allocation of the fixed factor land were examined. In the context of crop specific technologies, three equivalent models provide the possibility to alternatively estimate the parameters of profit-, cost- or production functions for each crop simultaneously while maintaining the assumed optimisation structure during estimation. Two extensions illustrated the generalisation to joint inputs in a multi-output framework and to explicitly incorporate risk behaviour of agricultural producers in an expected utility maximisation model. These derivations laid the ground to estimate structural parameters of programming models theoretically equivalent to the estimation of behavioural functions based on duality theory, but with the possibility to maintain more complex constrained optimisation structures that do not allow closed form solutions for input demand and output supply equations.

6) Exemplary applications of the specification approach demonstrated its functionality by evaluating estimation exercises based on multiple observations with Monte Carlo simulations. The 'Data constrained Generalised Maximum Entropy' approach (GME-D) was reviewed and employed as an estimation principle which allows to encompass situations with negative ('ill-posed') and positive ('well-posed') degrees of freedom. Monte Carlo simulation results for three different programming model structures, estimating parameters of a cost function, crop specific production functions, and crop specific profit functions indicate

consistent behaviour of the estimator in this context. The approach also proved its capability of estimating model parameters across binding and non-binding inequality constraints in the data generation process.

7) The GME-D approach also allows to address the problem of limited data information which often prevails in the context of differentiated, large scale modelling exercises. The inclusion of prior information in the form of supply elasticities and shadow prices of fixed resources was demonstrated and shown to improve small sample estimation accuracy. The employed reparameterisation of 'functions of parameters' is a very useful extension of previous GME-D applications that is closer to empirically relevant types of prior information. For one of the exemplatory programming models successful calibrations to base year observations were provided with different variants of prior information on supply elasticities. This approach relies on the same data constraints and generally allows to employ the same GME-D formalism as for well posed situations.

8) The final chapter of the study puts the estimation of programming models into the context of the large scale policy information system CAPRI. Results of an explorative application of this modelling system to evaluate agricultural policy changes in the 'grandes cultures' and beef sector implied by Agenda 2000 were presented. The modelling system combines EU-wide full coverage of the agricultural sector with a regional differentiated representation of agricultural production. Regional programming models allow to closely implement major policy measures of the CAP on the supply side. The integration with commodity markets at member state, EU and world level including relevant trade policy measures is realised by an iterative market clearing process. CAPRI thereby offers a comprehensive tool for the evaluation of EU agricultural policy programs. In addition, links to environmental indicators carry the analysis beyond the evaluation of effects related to agricultural commodities typically performed with agricultural sector models.

9) The specification of non-linear cost functions for CAPRI's regional programming models received special attention. The approach can be characterised as an application of PMP to multiple observations. The suggested methodology is designed to exploit information contained in a cross sectional sample to specify - regionally specific - quadratic cost functions with cross effects for crop activities. It constitutes the first application of PMP to multiple observations and also provides a solution to the 'curvature problem' with respect to the non-linear objective functions. The approach is applied to regional programming models for 22 regions in France. An ex-post simulation across the 1992 reform of the Common

Agricultural Policy shows plausible results with respect to the simulation behaviour of the resulting models, especially compared to a previously employed alternative PMP approach. A critical evaluation of the estimation methodology in light of the theoretical and methodological findings from the other parts of the study is given. This discussion points at extensions and improvements to be expected from the 'post PMP' approach to the estimation of programming models developed in this study.

We can conclude that the presented research was successful in providing theoretical and methodological groundwork for the estimation of programming models for agricultural supply analysis. It presented a framework which expanded the 'tool box' of the economic analyst, especially with respect to the specification of more complex supply specifications based on explicit constrained optimisation hypotheses. The parameters of these models can be quantified based on observed supply behaviour with statistical techniques.

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Zusammenfassung

Das übergeordnete Ziel der vorgelegten Arbeit war es, die methodische Lücke zwischen der Spezifikation beschrankter Optimierungsmodelle und dualen Systemen von Produktangebots- und Inputnachfragefunktionen bei der Abbildung von Multi-Output, Multi-Input Modellen landwirtschaftlichen Angebotsverhaltens zu schließen. Die Verfolgung dieses Ziels war von der Hoffnung getragen, die Flexibilität von Programmierungsmodellen durch differenzierte Berücksichtigung technologischer Beschrankungen und agrarpolitischer Instrumente mit der statistischen Spezifikation von Modellparametern zu verbinden. Vorgehensweise und Ergebnisse der Arbeit können durch die folgenden Punkte zusammengefaßt werden:

1) Die Positiv Mathematische Programmierung (PMP) – der bis heute am weitesten verbreitete Ansatz zur *Kalibrierung* von Programmierungsmodellen – wurde zunächst vorgestellt und bewertet. Die Einführung von zusätzlichen nicht-linearen Termen in der Zielfunktion auf Grundlage von Dualwerten von Kalibrierungsbeschrankungen erlaubt die Reproduktion des Angebotsverhalten eines Basisjahres und erzielt eine nahezu kontinuierliche Simulationsreaktion der resultierenden Modelle. In dieser Hinsicht ist der Ansatz linearen Programmierungsmodellen bei der Abbildung aggregierten Angebotsverhaltens überlegen. Allerdings führen die meisten Varianten dieser Methode zu willkürlichen quantitativen Anpassungsreaktionen, weil Informationen zu Anpassungsreaktionen fehlen und ad-hoc Annahmen getroffen werden, die eine Lösung des implizierten unterbestimmten Spezifikationsproblems der Parameter erlaubt. Es wurde argumentiert, dass eine zufriedenstellende Lösung nur durch die Einführung von a-priori Informationen, z.B. zu Angebotselastizitäten, oder die Verwendung mehrerer Beobachtungen gewährleistet werden kann.

2) Es folgte eine Bewertung der PMP Methode im Hinblick auf die Bereitstellung von Schätzgleichungen zur statistischen Parameterspezifikation mit mehreren Beobachtungen. Es konnte gezeigt werden, dass die strukturelle Diskrepanz zwischen der ersten Phase mit Kalibrierungsbeschrankungen und der resultierenden Modellspezifikation zu Schätzgleichungen führt, die keine konsistente Parameterschätzung unter der Annahme erlauben, dass das letztlich in Simulationen verwendete Modell eine gute Annäherung an den wahren Datengenerierungsprozess darstellt. Die Probleme ergeben sich aufgrund der inkonsistenten Festlegung von Dualwerten der Beschrankungen, die wiederum ein Fehlspezifikation der nichtlinearen Zielfunktionsterme mit sich bringen.

3) Ein einfacher, alternativer Ansatz zur Schätzung (und Kalibrierung) von Programmierungsmodellen wurde vorgestellt, der sich der notwendigen Bedingungen erster Ordnung des gewünschten Programmierungsmodells als Schätzgleichungen bedient. Dies erlaubt die simultane Schätzung von Modellparametern und Dualwerten von Beschrankungen mit einem statistischen Kriterium. Der Ansatz ist nicht nur für typische PMP Modelle geeignet, sondern kann auf allgemeine beschrankte Optimierungsmodelle angewendet werden. Er beinhaltet das Potential einer äquivalenten, aber zur gleichen Zeit flexibleren Methode zur Parameterschätzung ökonomischer Optimierungsstrukturen im Vergleich mit traditionellen ökonometrischen Schätzungen von Verhaltensfunktionen basierend auf der Dualitätstheorie.

4) Im Hinblick auf einen weiteren Kritikpunkt an typischen PMP Anwendungen und zur Herleitung korrespondierender Bedingungen erster Ordnung als Schätzgleichungen, wurden mögliche Rationalisierungen nicht-linearer Zielfunktionsterme unter den Hypothesen der Gewinnmaximierung bei Leontief Technologie betrachtet. Die Einführung produktspezifischer Landangebotsfunktionen zur Abbildung heterogener Landqualitäten erwies sich als konsistent zu PMP Modellen mit Kostenfunktionen, die nicht-linear in Aktivitätsumfängen sind. Eine weiterführende Betrachtung des zugrundeliegenden Aggregationsproblems bei Unkenntnis der Verteilung von Landqualitäten ergab eine Variante der 'convex combination constraints', wobei eine Umhüllungskurve aggregierter Extrempunkte durch eine nichtlineare Beschrankung angenähert werden kann. Die gleiche Spezifikation kann auch als Abbildung unbekannter Ressourcenbeschrankungen interpretiert werden.

5) Zur Demonstration der Anwendbarkeit des Schätzprinzips auf Modellstrukturen, die in der Vergangenheit Schätzungen von Verhaltensfunktionen zugrunde lagen, wurden Gewinnmaximierungsmodelle mit *allgemeinen* Technologieannahmen bei Allokation des fixen Faktors Land untersucht. Im Zusammenhang produktspezifischer Technologien wurden drei äquivalente Modelle vorgestellt, die alternativ die Schätzung der Parameter von Gewinn-, Kosten-, oder Produktionsfunktionen unter Einhaltung der unterstellten Optimierungsstruktur erlauben. Zwei Erweiterungen illustrieren Modelle mit Multi-Output Technologie und der Berücksichtigung von Risikoverhalten landwirtschaftlicher Produzenten im Rahmen einer Erwartungsnutzenmaximierung. Diese theoretischen Betrachtungen bilden die Basis für eine ökonometrische Spezifikation von Modellen, die äquivalent zu bisherigen Schätzungen von Verhaltensfunktionen sind, aber die Möglichkeit der Verwendung komplexerer Modellstrukturen beinhalten, die keine Auflösung zu Produktangebots- und Inputnachfragefunktionen erlauben.

6) Beispielhafte Anwendungen des Schätzprinzips zeigten dessen erfolgversprechende Funktionsweise durch Auswertung von Monte Carlo Simulationen wiederholter Schätzungen auf Basis mehrerer Beobachtungen. Der 'datenbeschrankte Verallgemeinerte Maximum Entropie Ansatz' (GME-D) wurde vorgestellt und als ökonometrisches Kriterium angewandt, weil es in Situationen mit positiven *und* negativen Freiheitsgraden nutzbar ist. Die Monte Carlo Simulationsergebnisse für drei verschiedene Strukturen von Programmierungsmodellen mit der Schätzung von Parametern einer Kostenfunktion, produktspezifischer Produktionsfunktionen und produktspezifischer Gewinnfunktionen deuten auf ein konsistentes Verhalten des Schätzers in diesem Zusammenhang hin. Der Ansatz zeigte sich auch in der Lage über bindende und nicht-bindende Nebenbedingungen des zugrundeliegenden Datengenerierungsprozesses hinweg zu schätzen.

7) Der GME-D Ansatz erlaubt darüber hinaus in eleganter Weise das – bei differenzierten Angebotsanalysen häufig beobachtete – Problem begrenzter Dateninformation anzugehen. Die Einführung von a-priori Information in Form von Angebotselastizitäten oder Schattenpreisen von Beschrankungen wurde erläutert und die potentiell positive Wirkung auf die Schätzgenauigkeit in kleinen Stichproben demonstriert. Die angewandte Reparametrisierung von *Funktionen* von Modellparametern stellt eine sehr nützliche Erweiterung vorheriger GME-D Anwendungen dar, die typischerweise verfügbare a-priori Informationen besser abbilden kann. Für eines der beispielhaften Programmierungsmodelle wurden erfolgreiche Kalibrierungen auf beobachtete Produktionsprogramme eines Basisjahres durchgeführt unter Verwendung verschiedener Varianten von Vorinformationen zu Angebotselastizitäten. Diese Vorgehensweise verwendet prinzipiell die selbe Gleichungsstruktur wie bei der Schätzung mit mehreren Beobachtungen.

8) Das abschließende Hauptkapitel der Arbeit setzt die Schätzung von Programmierungsmodellen in den Kontext des umfassenden agrarpolitischen Informationssystems CAPRI. Konzept und Ergebnisse einer explorativen Anwendung dieses Modellsystems auf die durch die Agenda 2000 eingeführten Politikänderungen im Bereich 'Grandes Cultures' und Rindfleisch wurden vorgestellt. Das Modellsystem kombiniert eine EU-weite Betrachtung des Agrarsektors mit regional differenzierter Abbildung landwirtschaftlicher Produktion. Regionale Programmierungsmodelle erlauben die differenzierte und genaue Abbildung der Politikinstrumente auf der Angebotsseite. Die Integration mit Produktmärkten auf nationaler, EU und internationaler Ebene unter Einbeziehung handelspolitischer Instrumente wird methodisch durch einen iterativen, markträumenden Prozess gewährleistet. Dadurch wird

CAPRI zu einem umfassenden Werkzeug zur Bewertung von agrarpolitischen Programmen der EU. Darüber hinaus erlaubt die Anbindung von Umweltindikatoren eine Analyse, die über die typische Aussagekraft von Agrarsektormodellen hinausgeht.

9) Vor dem Hintergrund der Zielsetzung dieser Arbeit erfährt natürlich die Spezifikation der nicht-linearen Kostenfunktionen für CAPRI's regionale Programmierungsmodelle besondere Aufmerksamkeit. Der realisierte Ansatz kann als Anwendung von PMP mit mehreren Beobachtungen verstanden werden. Die Methodik erlaubt die Ausnutzung von Informationen in einer Querschnittstichprobe zur Schätzung regionsspezifischer quadratischer Kostenfunktionen mit Kreuzeffekten zwischen pflanzlichen Produkten. Es stellt die erste Anwendung von PMP mit mehreren Beobachtungen dar und löst zusätzlich ein früheres Problem mit der Gewährleistung der richtigen Krümmungseigenschaften der Zielfunktion. Der Ansatz wird durch die Anwendung auf 22 französische Regionen illustriert. Eine ex-post Validierung simuliert das Modellverhalten über die Einführung der 1992er Reform der Gemeinsamen Agrarpolitik hinweg und zeigt plausibles Angebotsverhalten im Vergleich zu den Daten und einem alternativen PMP-Ansatz. Vor dem Hintergrund der theoretischen und methodischen Ergebnisse der grundlegenden Teile dieser Arbeit wird der Schätzansatz einer kritischen Evaluierung unterzogen. Diese Diskussion weist auf mögliche Erweiterungen und Verbesserungen hin, die aus den Erkenntnissen der präsentierten Forschung zu PMP-Alternativen erwachsen.

Abschließend ist zu resümieren, dass es der vorliegenden Forschungsarbeit gelungen ist eine theoretische und methodische Grundlage zur Schätzung von Programmierungsmodellen in der Agrarangebotsanalyse zu schaffen. Der definierte Rahmen erweitert den 'Werkzeugkasten' des angewandten ökonomischen Analytikers bei der Verwendung komplexer Angebotsspezifikationen auf Basis expliziter, beschrankter Optimierungsmodelle. Die Parameter dieser Modelle können unter Verwendung beobachteten Angebotsverhaltens mit statistischen Kriterien quantifiziert werden.

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